The Same Ten Per Cent Rise Both Raises and Lowers Revenue, and There Is Only One Boundary

— If demand goes as the $-e$ power of price, raising the price by 1.1 multiplies revenue by 1.1^{1-e} — the separator is whether the elasticity crosses one — [Paper 698]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

Abstract

Whether a price rise raises or lowers revenue is not settled by the size of the rise. It is settled by one exponent, how strongly demand answers price.That the boundary sits exactly at one is put into numbers here.No new theorem or law is claimed. First, and this is the core. The boundary sits exactly at $e = 1$——there, moving the price by any amount leaves revenue at 1.000000000 (Section 2). **Second, below the boundary it rises and above it falls——for a rise of 1.1, $+0.079230$ at $e = 0.2$, $+0.048809$ at $e = 0.5$, -0.090909 at $e = 2$ and -0.173554 at $e = 3$** (Section 2). **Third, the size has nothing to do with the sign——a rise of only 1.01 still gives $+0.004988$ at $e = 0.5$ and -0.009901 at $e = 2$** (Section 3). **Fourth, a price cut does not move the boundary——cutting to $1/1.1$ gives -0.046537 at $e = 0.5$ and $+0.100000$ at $e = 2$, only reversing the direction** (Section 3). **Fifth, so something is settled before "by how much"——without knowing which side of one the elasticity lies, no discussion of the size gives the sign** (Section 3). **Sixth, the separator is whether the elasticity crosses one——the dividing line lies not in any quantity but in the exponent** (Section 4). **Keywords**: price elasticity, unit elasticity, revenue and price, constant-elasticity demand, sign boundary, role of the exponent.

Scope of this paper (scope note): **No new theorem or law is claimed——price elasticity, the stationarity of revenue at unit elasticity and constant-elasticity demand are all standard (Marshall 1890, Varian 2014).No elasticity is claimed ——e is taken as given, and no figure is offered for any real good.The form of demand is fixed——only the constant-elasticity form is used. With linear demand the elasticity varies along the curve and the boundary is no longer a single point.Profit is not discussed——costs, profit and the optimal price are not treated; only revenue is examined.Estimation is not treated——how an elasticity is measured, and with what error, is not the subject.Relation to earlier papers**: **Paper 558 showed that three different powers live in one formula for a spring, so that the same "ten per cent" becomes 1.464100 on the wire diameter, 0.751315 on the coil diameter and 0.909091 on the number of turns ——there the exponents were given by the formula and the question was which dimension to move. Here there is only one exponent, it cannot be known without measuring, and crossing one changes the sign.**What is added is **showing the boundary to sit at $e = 1$ with revenue exactly invariant there**, **tabulating the factor against e**, **showing the sign to be independent of the size of the change**, **showing a price cut to leave the boundary fixed**, and **placing the separator on whether the elasticity crosses one**.

1. Notation and the Question

Setting: **let demand go as the $-e$ power of price. Revenue is price times quantity and therefore goes as the $1 - e$ power of price.**

$$Q \propto P^{-e}, \qquad R = PQ \propto P^{\,1-e}, \qquad \frac{R'}{R} = \left(\frac{P'}{P}\right)^{1-e}$$

Call e the elasticity. At $e = 1$ the exponent is zero.

Question: **what settles whether a rise raises or lowers revenue: the size, or the exponent?**

2. The Boundary Sits at $e = 1$

Computation 2.1: **the factors in revenue and quantity on raising the price by 1.1 were derived for each e.**

e	revenue	quantity
0.0	1.100000	1.000000
0.2	1.079230	0.981118
0.5	1.048809	0.953463
1.0	1.000000	0.909091
1.5	0.953463	0.866784
2.0	0.909091	0.826446
3.0	0.826446	0.751315

Observation 2.2

this is the core: at $e = 1$ revenue does not depend on price at all.The exponent is zero, so doubling or halving the price leaves revenue at 1.000000000——not approximately, but as an identity.

Observation 2.3

quantity falls at every e.Even at $e = 0.2$ the quantity falls to 0.981118——where revenue rises, it is not that more is sold, only that the fall has not kept up with the rise in price.

3. The Size Has Nothing to Do with the Sign

Computation 3.1: the change in revenue was derived for several sizes of rise at two values of e.

price	$e = 0.5$	$e = 2$
1.01	$+0.004988$	-0.009901
1.10	$+0.048809$	-0.090909
1.50	$+0.224745$	-0.333333
2.00	$+0.414214$	-0.500000

Observation 3.2

the size changes only the magnitude; the exponent alone fixes the sign.Even a rise of 1.01 gains at $e = 0.5$ and loses at $e = 2$——“let us try raising it just a little” is in no way cautious about the sign.

Computation 3.3: cutting the price to $1/1.1$ gives -0.046537 at $e = 0.5$ and $+0.100000$ at $e = 2$.

Observation 3.4

the boundary does not move; only the direction reverses.Below one, raising gains and cutting loses; above one, the reverse——which way to move the price is again settled by which side of one the elasticity lies.

Observation 3.5

so the order of the discussion is fixed.Until which side of one is settled, no amount of care over the size produces a sign——arguing about the size belongs after the sign is known.

4. The Separator

Observation 4.1

the separator is whether the elasticity crosses one.Not price, not quantity, not revenue: the dividing line lies in the exponent——$e = 1$ is not the border between “responsive” and “unresponsive” but the single point at which the sign of the change in revenue turns over.

Observation 4.2

beside Paper 558.Paper 558 showed three different powers living in one formula for a spring, the same “ten per cent” becoming 1.464100, 0.751315 or 0.909091——there the exponents came from the formula and the question was which dimension to move. Here there is one exponent, and it cannot be known without measuring. The same phrase, “the same ten per cent gives different answers”, with the dividing line in a different place.

Observation 4.3

what is not said.No real elasticity, no cost, no profit and no estimation is treated, and demand is fixed to the constant-elasticity form——with linear demand the elasticity varies along the curve and the boundary is no longer a point.

5. Roll Call and Closing

1. **exactly $\mathbf{1.000000000}$ at $e = 1$** (Computation 2.1): **the core.**
2. **$+0.079230$ at $e = 0.2$** (Computation 2.1): **below the boundary.**
3. **-0.173554 at $e = 3$** (Computation 2.1): **above it.**
4. **quantity falls at every e** (Observation 2.3): **0.981118 even at $e = 0.2$.**
5. **$+0.004988$ and -0.009901 at 1.01** (Computation 3.1): **size does not touch the sign.**
6. **$+0.414214$ and -0.500000 at 2.00** (Computation 3.1): **only the magnitude grows.**
7. **-0.046537 and $+0.100000$ on a cut** (Computation 3.3): **the direction reverses.**
8. **the boundary does not move** (Observation 3.4): **still $e = 1$.**
9. **the order is fixed** (Observation 3.5): **sign before size.**
10. **does e cross one** (Observation 4.1): **the separator; Paper 558.**

Closing: **The same ten per cent rise both raises and lowers revenue.The boundary sits exactly at $e = 1$, where moving the price by any amount leaves revenue at 1.000000000——an identity, not an approximation.Below it revenue rises and above it falls: $+0.079230$ at $e = 0.2$ and -0.173554 at $e = 3$ for a rise of 1.1.Changing the size of the rise from 1.01 to 2.00 never changes the sign; the size changes only the magnitude——"let us try raising it just a little" is in no way cautious about the sign.A price cut leaves the boundary where it is and only reverses the direction.The separator is whether the elasticity crosses one.Placed among the earlier papers——in Paper 558 three exponents came from the formula and the question was which dimension to move; here there is one exponent and it cannot be known without measuring.To be honest——demand is fixed to the constant-elasticity form, and with linear demand the boundary is no longer a point.**

References

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Yuuki YAMAGISHI (山岸 勇輝)

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