Dividing Does Not Always Make It Faster: What Decides Is Not the Division but the Combining

— For $T(n) = aT(n/b) + f(n)$ the exponent of the dividing side is $\log_b a$ — the separator is which of the two sides governs — [Paper 702]

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— *From thinking to syncing.* —

Abstract

Divide and conquer is explained as a way of going faster. But if the work of putting the pieces together is heavy, the division has gained nothing.Which side governs, and when an exponent gap beats a constant factor, are counted here.No new theorem or law is claimed. First, and this is the core. Dividing sometimes gains nothing——with $a = 2$ and $b = 2$, combining work of n gives $n \log n$, while combining work of n^2 leaves n^2 (Section 2). **Second, what governs is only which of two exponents is larger——compare $\log_b a$ with the exponent of the combining work; the larger is the answer, and only when they are equal does a logarithm appear** (Section 2). **Third, even where there is a gain, it is small——Strassen's exponent 2.807355 lies below the naive 3.000000 by only 0.192645** (Section 3). **Fourth, that gap struggles against a constant factor——a factor of merely 2 against it postpones the gain to $n > 36.527043$, a factor of 5 to $n > 4248.919974$ and a factor of 20 to $n > 5669014.762747$** (Section 3). **Fifth, a larger gap changes the story——Karatsuba's exponent is 1.584963, a gap of 0.415037, so even a factor of 2 is overtaken at $n > 5.312610$** (Section 3). **Sixth, the separator is which side governs——not how cleverly the problem is divided: if the combining exponent is larger, every division gives the same answer** (Section 4). **Keywords**: divide and conquer, master theorem, exponent of complexity, constant factor, crossing point, asymptotic against practical.

Scope of this paper (scope note): **No new theorem or law is claimed——the three cases of the master theorem and the exponents of Strassen and Karatsuba are all standard (Cormen et al. 2009, Knuth 1997).No implementation is measured——actual speed is not measured, and the constant factor is a representative value taken as given.The best known exponent is not treated——the best exponent currently known for matrix multiplication, near 2.4, is mentioned and not treated.Machine matters are not treated——memory hierarchy, caches and parallelism are left out.Stability is not discussed——the growth of numerical error in Strassen's method is not the subject.Relation to earlier papers**: **Paper 681 showed that the diameter of a crater grows only as the $1/3.4$ power of the energy, and that the gap of 0.039216 between the gravity exponent 0.294118 and the strength exponent $1/3$ opens with the decades extrapolated ——there an exponent gap acted as the risk of extrapolation; here the same kind of gap decides when a constant factor is overtaken. Not extrapolation but a crossing point.**What is added is **setting the three cases side by side as a comparison of exponents**, **showing that where the combining side governs the division gains nothing**, **obtaining crossing points from the gap 0.192645 and a constant factor**, **showing the crossing points of Karatsuba and Strassen to differ by an order**, and **placing the separator on which side governs**.

1. Notation and the Question

Setting: **cut a problem of size n into b parts, solve a of them, and combine with work $f(n)$.**

$$T(n) = aT\left(\frac{n}{b}\right) + f(n), \qquad e_{\text{divide}} = \log_b a, \qquad f(n) = n^c$$

If $e_{\text{divide}} > c$ the dividing side governs, if $e_{\text{divide}} < c$ the combining side does, and if they are equal a logarithm appears.

Question: **when does dividing actually make it faster?**

2. Which Side Governs

Computation 2.1: **the two exponents were compared for representative cases.**

a, b, f	$\log_b a$	governed by	answer
$2, 2, n$	1.000000	balance	$n \log n$
$2, 2, n^2$	1.000000	combining	n^2
$8, 2, n^2$	3.000000	dividing	n^3
$7, 2, n^2$	2.807355	dividing	$n^{2.807355}$
$3, 2, n$	1.584963	dividing	$n^{1.584963}$
$4, 2, n$	2.000000	dividing	n^2

Observation 2.2

this is the core: in the second row the division has gained nothing.Dividing with $a = 2$ and $b = 2$ still leaves n^2 when the combining work is n^2, the same exponent as not dividing at all——"we divided it, so it must be faster" is not looking at the combining.

Observation 2.3

the sixth row says the same from the other side.With $a = 4$ and $b = 2$ the dividing exponent is 2.000000, the same as naive multiplication——divide into too many pieces and the dividing itself becomes the loss.

Observation 2.4

only the balanced case brings a logarithm.The $n \log n$ of merge sort follows from the two exponents being exactly equal——a logarithm is the mark of neither side governing.

3. An Exponent Gap Struggles Against a Constant Factor

Computation 3.1: **Strassen's exponent is 2.807355, below the naive 3.000000 by 0.192645. The n at which it overtakes a constant-factor handicap was derived.**

constant-factor handicap	overtakes at
2	36.527043
5	4248.919974
20	5669014.762747

> **Observation 3.2**
>
> a constant handicap acts through the reciprocal of the exponent gap.Raising the handicap from 2 to 5, a factor of 2.5, moves the crossing from 36.527043 to 4248.919974, two orders——the smaller the exponent gap, the heavier the price of a constant factor.

Computation 3.3: Karatsuba's exponent is 1.584963, below the naive 2 by 0.415037; a handicap of 2 is overtaken at $n > 5.312610$.

> **Observation 3.4**
>
> two methods that both "lower the exponent" become useful an order apart.Karatsuba from 5.312610, Strassen from 36.527043——what matters is not how far the exponent fell but whether the fall is worth the constant factor.

> **Observation 3.5**
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> so "asymptotically faster" is no guarantee of being usable.Under a handicap of 20 the crossing is at $n > 5669014.762747$, and if the sizes handled never reach that, it is never once faster——the exponent is a statement about infinity and the constant is a statement about what is in front of you.

4. The Separator

Observation 4.1

the separator is which of the two sides governs.Not how cleverly the problem is divided: where the combining exponent is larger, every division gives the same answer——the room for improvement lies on the side of combining, not dividing.

Observation 4.2

beside Paper 681.Paper 681 showed the diameter of a crater to grow only as the $1/3.4$ power of the energy, the gap of 0.039216 between 0.294118 and $1/3$ opening with the decades extrapolated——there an exponent gap acted as the risk of extrapolation; here the same kind of gap decides when a constant factor is overtaken.

Observation 4.3

what is not said.Implementation, machine matters and stability are not treated, and the constant factor is representative——what is shown is the comparison of exponents and where a gap beats a constant.

5. Roll Call and Closing

1. $a = 2, b = 2, f = n$ **balances** (Computation 2.1): **giving $n \log n$.**
2. $f = n^2$ **still gives** n^2 (Computation 2.1): **the core; the division is wasted.**
3. $a = 4, b = 2$ **also gives** n^2 (Computation 2.1): **dividing too far is a loss.**
4. **Strassen at** $\mathbf{2.807355}$ (Computation 3.1): **below the naive** $\mathbf{3}$**.**
5. **a gap of** $\mathbf{0.192645}$ (Computation 3.1): **small.**
6. **a handicap of** 2 **needs** $n > \mathbf{36.527043}$ (Computation 3.1): **gain begins here.**
7. **a handicap of** 5 **needs** $n > \mathbf{4248.919974}$ (Computation 3.1): **two orders away.**
8. **a handicap of** 20 **needs** $n > \mathbf{5669014.762747}$ (Computation 3.1): **it may never arrive.**
9. **Karatsuba at** $n > \mathbf{5.312610}$ (Computation 3.3): **a gap of** $\mathbf{0.415037}$**.**
10. **which side governs** (Observation 4.1): **the separator; Paper 681.**

Closing: **Dividing does not always make it faster.With $a = 2$ and $b = 2$, combining work of n^2 leaves the answer at n^2, the same exponent as not dividing at all——what governs is only which of $\log_b a$ and the combining exponent is larger, and only equality brings a logarithm.Even where there is a gain it is small——Strassen's 2.807355 lies below the naive 3.000000 by only 0.192645.So a constant factor of merely 2 postpones the gain to $n > 36.527043$, a factor of 5 to $n > 4248.919974$ and a factor of 20 to $n > 5669014.762747$——the exponent is a statement about infinity and the constant is a statement about what is in front of you.With a gap of 0.415037, Karatsuba overtakes a factor of 2 already at $n > 5.312610$.The separator is which of the two sides governs.Placed among the earlier papers——in Paper 681 an exponent gap acted as the risk of extrapolation; here the same gap decides a crossing point.To be honest——implementation and machine matters are not treated, and the constant factor is representative.**

References

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