Even at a Hit Rate of 0.99, More Than Half the Time Is Spent on Misses

— at **0.99** the share of time taken by misses is **0.502513**: an event of only **0.01** carries away more than half — the separator is whether the product of the minority share and the ratio exceeds one — [Paper 705]

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— From thinking to syncing. —

Abstract

A hit rate of 99 per cent sounds like almost everything hitting. But if the slow side is a hundred times slower, more than half the time is spent on that one per cent.Which side governs is counted here as the product of a share and a ratio.No new theorem or law is claimed. First, and this is the core. At a hit rate of 0.99, misses take 0.502513 of the time——an event of only 0.01 carries away more than half (Section 2). **Second, the average is nearly twice the ideal ——at 0.99 it is 1.990000 cycles, close to twice what it would be with no misses at all** (Section 2). **Third, improvement falls off sharply——0.90 to 0.99 gives 5.477387, while 0.99 to 0.999 gives 1.810737 and 0.999 to 0.9999 only 1.088227** (Section 3). **Fourth, it falls off because the governing side changes——misses stop governing where the product of share and ratio falls below one, exactly at a hit rate of 0.990000** (Section 3). **Fifth, so a hit rate on its own says nothing——the same 0.99 gives 1.090000 cycles if the slow side is ten times slower, 1.990000 at a hundred and 10.990000 at a thousand** (Section 3). **Sixth, the separator is whether the product of the minority share and the ratio exceeds one——neither the share nor the ratio alone decides which side governs** (Section 4). **Keywords**: cache hit rate, average access time, locality of reference, Amdahl's law, governing term, diminishing improvement.

Scope of this paper (scope note): **No new theorem or law is claimed——the average access time, locality of reference and Amdahl's law are all standard (Hennessy and Patterson 2019, Amdahl 1967).Organisation is not treated——hierarchy, replacement policy, associativity and write policy are not treated.The origin of locality is not discussed——why programs exhibit locality is not the subject.The figures are not claimed——one cycle for a hit and a hundred for a miss are representative and move greatly between machines.The general law is not entered——neither the derivation of Amdahl's law nor the limits of parallelism is treated; only the sameness of the form is noted.Relation to earlier papers**: **Paper 702 showed that whether divide and conquer is faster is decided by which of $\log_b a$ and the combining exponent is larger——there two asymptotic exponents competed and the answer was settled at infinity. Here a finite share and a finite ratio compete, and the boundary lands on a number in front of you.**What is added is **tabulating the average and the share of time taken by misses**, **showing that share to exceed half at $\mathbf{0.502513}$**, **putting the falloff of improvement at $\mathbf{5.477387}$, $\mathbf{1.810737}$ and $\mathbf{1.088227}$**, **showing the boundary to sit exactly at a hit rate of $\mathbf{0.990000}$**, and **placing the separator on the product of share and ratio**.

1. Notation and the Question

Setting: **a hit costs t_h cycles and a miss costs t_m. With a hit rate h, the average and the share of time taken by misses follow.**

$$\bar{t} = h\,t_h + (1-h)\,t_m, \qquad \phi = \frac{(1-h)\,t_m}{\bar{t}}, \qquad (1-h)\frac{t_m}{t_h} < 1$$

Take $t_h = 1$ and $t_m = 100$. The condition on the right is where misses stop governing.

Question: **what does "a hit rate of 99 per cent" actually say?**

2. One Per Cent Takes More Than Half the Time

Computation 2.1: **the average and the share of time taken by misses were derived for each hit rate.**

hit rate	average	share taken by misses
0.9000	10.900000 cycles	0.917431
0.9900	1.990000 cycles	0.502513
0.9990	1.099000 cycles	0.090992
0.9999	1.009900 cycles	0.009902

Observation 2.2

this is the core: an event of 0.01 takes 0.502513 of the time.Counted by occasions it is one per cent; counted by time it is more than half——without saying what is being counted, the same 0.99 gives opposite impressions.

Observation 2.3

the average is nearly twice the ideal.At a hit rate of 0.99 the average of 1.990000 cycles is close to twice the value with no misses——"almost everything hits" and "almost as good as ideal" are different statements.

3. Where Improvement Falls Off

Computation 3.1: **the speedup from raising the hit rate was derived: 5.477387 from 0.90 to 0.99, 1.810737 from 0.99 to 0.999 and 1.088227 from 0.999 to 0.9999.**

Observation 3.2

the same effort of removing one decade of misses buys less at every decade.The identical task falls from 5.477387 to 1.088227——improvement is worth only as much as the governance that remains.

Computation 3.3: **misses stop governing where the product of share and ratio falls below one, exactly at a hit rate of 0.990000.**

Observation 3.4

the boundary lands exactly on the second row of the table.At 0.99 the governance passes from the misses to the hits——improvement falls off not for want of effort but because the governing side has changed.

Observation 3.5

so a hit rate on its own says nothing.The same 0.99 gives an average of 1.090000 cycles if the slow side is ten times slower, 1.990000 at a hundred and 10.990000 at a thousand——hit rate and ratio mean something only as a pair.

4. The Separator

Observation 4.1

the separator is whether the product of the minority share and the ratio exceeds one.A small share governs if the ratio is large enough, and a large ratio does not govern if the share is small enough——neither one alone licenses “that is good enough”.

Observation 4.2

beside Paper 702.Paper 702 showed that whether divide and conquer is faster is decided by which exponent is larger——there two asymptotic exponents competed and the answer was settled at infinity; here a finite share and a finite ratio compete and the boundary lands at a hit rate of 0.990000, a number in front of you. The same question of governance, settled in a different place.

Observation 4.3

what is not said.Organisation and the origin of locality are not treated, and the cycle counts are representative——what is shown is only that the product of share and ratio decides which side governs.

5. Roll Call and Closing

1. **1.990000 cycles at 0.99** (Computation 2.1): **nearly twice the ideal.**
2. **misses take 0.502513** (Computation 2.1): **the core.**
3. **10.900000 at 0.90** (Computation 2.1): **misses govern entirely.**
4. **1.009900 at 0.9999** (Computation 2.1): **at last near ideal.**

5. **5.477387 from 0.90 to 0.99** (Computation 3.1): **it works.**
6. **1.810737 from 0.99 to 0.999** (Computation 3.1): **falling off.**
7. **1.088227 from 0.999 to 0.9999** (Computation 3.1): **almost nothing.**
8. **the boundary is 0.990000** (Computation 3.3): **governance changes.**
9. **10.990000 at a ratio of a thousand** (Observation 3.5): **the rate alone is not enough.**
10. **share times ratio** (Observation 4.1): **the separator; Paper 702.**

Closing: **Even at a hit rate of 0.99, more than half the time is spent on misses.With a hit costing 1 cycle and a miss 100, the average at 0.99 is 1.990000 cycles and the share of time taken by misses is 0.502513——counted by occasions it is one per cent; counted by time it is more than half.Improvement falls off sharply, 5.477387 from 0.90 to 0.99 becoming 1.810737 and then 1.088227.It falls off not for want of effort: misses stop governing exactly at a hit rate of 0.990000, and there the governing side changes——the same 0.99 gives 10.990000 cycles if the slow side is a thousand times slower.Hit rate and ratio mean something only as a pair.The separator is whether the product of the minority share and the ratio exceeds one.Placed among the earlier papers——in Paper 702 two asymptotic exponents competed and the answer was settled at infinity; here the boundary lands on a number in front of you.To be honest——organisation and the origin of locality are not treated, and the cycle counts are representative.**

References

1. Amdahl, G. M. (1967). Validity of the single processor approach to achieving large scale computing capabilities. *AFIPS Conference Proceedings*.
2. Hennessy, J. L., & Patterson, D. A. (2019). *Computer Architecture: A Quantitative Approach*. Morgan Kaufmann.
3. Yamagishi, Y. (2026). *Dividing Does Not Always Make It Faster: What Decides Is Not the Division but the Combining* [Paper 702]. Zenodo. 10.5281/zenodo.23014909

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