Nobody’s Expectation Enters the Price of a Future

— expecting the asset to rise by 20 % or to fall by 20 % leaves F unmoved, the largest and smallest differing by less than 10^{-12} — the separator is whether no-arbitrage is pinning the price to a point or friction is loosening it to a band — [Paper 709]

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— *From thinking to syncing.* —

Abstract

Prices are said to come from supply and demand. Yet the price of a future is fixed without using anybody’s expectation of anything.What enters and what does not is checked here, in the formula and in the profit.No new theorem or law is claimed. First, only the interest rate enters——with $S = 100$ and $T = 1$ year, 102.020134 at 2 %, 105.127110 at 5 % and 110.517092 at 10 % (Section 2). **Second, and this is the core. Expectations do not enter——varying the expected return by ± 20 % leaves the largest and smallest values of F differing by less than 10^{-12}** (Section 2). **Third, that they do not enter can be checked in the profit——mispriced by 1, the arbitrage profit is exactly 1.000000 whether the price a year later is 60 or 150** (Section 2). **Fourth, the price is a point only without friction——a transaction cost of 0.5 % each way turns F into a band from 104.627110 to 105.627110** (Section 3). **Fifth, the width of the band is proportional to the friction——1.000000 at 0.5 % each way, and 2.000000 times that at 1 %** (Section 3). **Sixth, the separator is no-arbitrage pinning against friction loosening——expectations return only inside the band** (Section 4). **Keywords**: no-arbitrage, forward price, expected return, transaction costs, no-arbitrage band, quantities that do not act.

Scope of this paper (scope note): **No new theorem or law is claimed——the determination of a forward price by no-arbitrage, and the band produced by transaction costs, are both standard (Hull 2018, Cox and Rubinstein 1985).No level is claimed——the interest rate and the price are assumed representative values; nothing is said about any real market.Only a simple asset is treated——dividends, storage costs and convenience yields are left out; continuous compounding is used and day-count conventions are not entered.Practice is not treated——short-selling constraints, margin and settlement are not treated; only the single fact that friction produces a band is used.Inside the band is not discussed——where within the band the price settles is not fixed by no-arbitrage, and what governs it there is outside this paper.Relation to earlier papers**: **Paper 430 showed that widening the contact a hundredfold does not change the friction, naming a quantity that does not act, the apparent area of contact——there the inert quantity was geometric. Here it is a human expectation. The same gesture of naming what does not act, applied outside physics.**What is added is **tabulating the forward price against the interest rate**, **checking numerically that varying the expected return leaves F unmoved**, **showing the arbitrage profit to be exactly 1.000000 whatever the future price**, **showing friction to turn a point into a band of width 1.000000**, and **placing the separator on no-arbitrage against friction**.

1. Notation and the Question

Setting: **let F be the agreed price for delivery in T years of an asset worth S today. If borrowing to buy and carry does not cost the same as buying the future, a riskless profit is available.**

$$F = Se^{rT}, \qquad \left[Se^{rT} - cS,\ Se^{rT} + cS\right], \qquad \text{width} = 2cS$$

Here c is the transaction cost each way. Take $S = 100$ and $T = 1$ year.

Question: **whose expectation enters this price?**

2. Only the Interest Rate Enters

Computation 2.1: **the forward price was derived for each interest rate.**

interest rate	forward price
2 %	102.020134
5 %	105.127110
10 %	110.517092

Computation 2.2: the forward price was written as a function taking the expected return as an argument, and evaluated at $0\ \%$, $+20\ \%$ and $-20\ \%$. The largest and smallest of the three differed by less than 10^{-12}.

> **Observation 2.3**
>
> this is the core: the price is fixed without anybody's expectation.Whether the buyer is bullish or bearish, F is the same 105.127110——there are quantities of which "it is set by supply and demand" is simply not true.

Computation 2.4: suppose the future is priced 1 too high. Sell the future, borrow, buy the asset and carry it. Whatever the price a year later, 1.000000 remains.

price after one year	arbitrage profit
60	1.000000
100	1.000000
150	1.000000

> **Observation 2.5**
>
> "it does not enter" can be checked in this table.The profit is settled without knowing how the price will move, so no expectation of that price was used in fixing the price——that something does not act can be shown in a balance sheet rather than in words.

3. A Point Only Without Friction

Computation 3.1: with a transaction cost of $0.5\ \%$ each way, the range over which no arbitrage is available runs from 104.627110 to 105.627110, a width of 1.000000.

> **Observation 3.2**
>
> friction does not remove the answer; it turns the answer into a band.With costs, a small deviation is not worth exploiting, so the price is no longer pinned to a point——between "determined" and "undetermined" lies "determined only within a band".

Computation 3.3: **the width is $2cS$, so doubling the cost to $1\ \%$ makes it 2.000000 times as wide.**

> **Observation 3.4**
>
> **so the strength of "fixed by no-arbitrage" can be measured by the friction.The smaller the friction, the narrower the band and the stronger the claim——the strength of a theory is set by the plumbing of a market.**

> **Observation 3.5**
>
> **and inside the band, expectations return.No-arbitrage speaks only outside the band, so where within it the price settles is another matter——expectations are not inert; the place where they can act is confined to the band.**

4. The Separator

> **Observation 4.1**
>
> **the separator is whether no-arbitrage is pinning the price to a point or friction is loosening it to a band.No-arbitrage squeezes the price by saying that anything else would be free money; friction relaxes that squeeze ——the two act in opposite directions, and the difference is visible as the width of the band.**

> **Observation 4.2**
>
> **beside Paper 430.Paper 430 showed that widening the contact a hundredfold does not change the friction, naming a quantity that does not act——there the inert quantity was geometric; here it is a human expectation. Naming what does not act works outside physics too.**

> **Observation 4.3**
>
> **what is not said.Dividends, storage and practice are not treated, and nothing is said about what governs the price inside the band——what is shown is which quantities enter, which do not, and that friction makes a band.**

5. Roll Call and Closing

1. **102.020134 at 2 %** (Computation 2.1): **what enters.**
2. **105.127110 at 5 %** (Computation 2.1): **likewise.**
3. **110.517092 at 10 %** (Computation 2.1): **higher with the rate.**
4. **expectations move it by under 10^{-12}** (Computation 2.2): **the core.**
5. **profit 1.000000 at a price of 60** (Computation 2.4): **checked in the balance.**
6. **the same at 150** (Computation 2.4): **independent of the future.**
7. **band 104.627110 to 105.627110** (Computation 3.1): **once friction enters.**
8. **width 1.000000** (Computation 3.1): **at 0.5 % each way.**
9. **doubling the cost doubles it** (Computation 3.3): **2.000000 times.**
10. **no-arbitrage or friction** (Observation 4.1): **the separator; Paper 430.**

Closing: **Nobody's expectation enters the price of a future.With $S = 100$ and $T = 1$ year the price is 102.020134 at 2 %, 105.127110 at 5 % and 110.517092 at 10 %: only the interest rate enters.Varying the expected return by ± 20 % leaves the largest and smallest values differing by less than 10^{-12}——and that can be checked in the balance: mispriced by 1, the arbitrage profit is exactly 1.000000 whether the price a year later is 60 or 150.The price is a point only without friction: a cost of 0.5 % each way makes it a band from 104.627110 to 105.627110, of width 1.000000——and the width is proportional to the cost, 2.000000 times as wide at 1 %.Inside that band, expectations return.The separator is whether no-arbitrage is pinning the price to a point or friction is loosening it to a band.Placed among the earlier papers——in Paper 430 the inert quantity was geometric; here it is a human expectation.To be honest——dividends and practice are not treated, and nothing is said about the price inside the band.**

References

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