Return Grows with Size and Cost Grows with Size to the Power 1.5, So an Optimal Size Must Exist

— At **5** bp each way and **252** turnovers a year, the annual cost is **0.252000** — the separator is the power with which cost grows with size — [Paper 710]

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— *From thinking to syncing.* —

Abstract

A good strategy is supposed to earn more the bigger it is run. But if cost grows as the 1.5 power of size, it must turn over somewhere.That the turning point is set by the difference of the powers alone is counted here.No new theorem or law is claimed. First, turnover alone is already a large expense——5 bp each way and 252 turnovers a year give an annual cost of 0.252000 (Section 2). **Second, so the gross performance demanded leaps——at a volatility of 15 %, a net Sharpe ratio of 1.0 demands 2.680000 gross** (Section 2). **Third, increasing size increases cost faster than size——with cost per unit going as the square root of size, a hundredfold size gives 1000.000000 times the total cost** (Section 3). **Fourth, and this is the core. Powers of 1 and 1.5 force an optimal size to exist——with a return of 0.10×size and a cost of 0.01×size$^{1.5}$, the optimum is 44.444444 with a net of 1.481481** (Section 3). **Fifth, the shape of the turn is set by the difference of the powers, not by the coefficients——the net returns to zero at exactly 2.250000 times the optimum, and that ratio does not move with the coefficients** (Section 3). **Sixth, the separator is the power with which cost grows with size——at a power of 1 there is no optimum; the moment it exceeds 1, one appears** (Section 4). **Keywords**: transaction costs, market impact, optimal size, capacity, difference of exponents, turnover.

Scope of this paper (scope note): **No new theorem or law is claimed——the accumulation of transaction costs, the square-root rule of thumb for market impact and limits on capacity are all standard (Almgren and Chriss 2000, Lopez de Prado 2018).Nothing particular is said——no claim is made about any fund, product or person.The figures are not claimed——5 bp each way, 252 turnovers and a volatility of 15 % are representative and move greatly with the strategy.The power is not derived——the exponent $1/2$ for market impact is taken as a representative value; measurements range over roughly 0.4 to 0.6.Execution is not discussed——order splitting, spreading over time and the choice of algorithm are not treated.Relation to earlier papers**: **Paper 335 showed that making a round trip changes the exponent, radar turning a power of two into a power of four in distance——there the exponent of one quantity was changed by an operation. Here two quantities of different exponents are added, and a difference of only 0.5 forces an interior optimum.**What is added is **putting the cost of turnover at $\mathbf{0.252000}$ a year**, **obtaining the gross Sharpe ratio of $\mathbf{2.680000}$ demanded**, **separating the effect of size into per unit and total**, **giving the optimal size $\mathbf{44.444444}$ and showing the zero to sit at exactly $\mathbf{2.250000}$ times it**, and **placing the separator on the power with which cost grows**.

1. Notation and the Question

Setting: **let b be the cost each way and n the number of turnovers a year. Against size Q, let the return be proportional and the cost of market impact grow as $Q^{1.5}$.**

$$C_{\text{year}} = 2bn, \qquad \Pi(Q) = aQ - \beta Q^{3/2}, \qquad Q^{*} = \left(\frac{2a}{3\beta}\right)^{2}, \qquad Q_{0} = \left(\frac{a}{\beta}\right)^{2}$$

Q^{*} is the size at which the net is greatest and Q_{0} the size at which it returns to zero.

Question: **does the same strategy earn more the bigger it is run?**

2. Turnover Alone Is an Expense

Computation 2.1: **at 5 bp each way and 252 turnovers a year, the annual cost is 0.252000.**

Computation 2.2: **at a volatility of 15 %, the gross Sharpe ratio demanded for each net Sharpe ratio follows.**

net Sharpe ratio	gross Sharpe ratio demanded
0.5	2.180000
1.0	2.680000
1.5	3.180000

Observation 2.3

the cost is added on top by the same amount whatever the target.The rows differ by 0.500000 each, and the cost contributes a flat $\frac{0.252000}{0.15} = 1.680000$——reducing turnover sometimes helps more than searching for a slightly better strategy.

Observation 2.4

a gross of 2.680000 is rarely seen.As Paper 706 showed, even with no skill at all the best of a thousand attempts reaches 1.449610——and 2.680000 is far beyond that.

3. Cost Grows Faster Than Size

Computation 3.1: with impact going as the square root of size, size acts differently per unit and in total.

size	cost per unit	total cost
1	1.000000	1.000000
10	3.162278	31.622777
100	10.000000	1000.000000

Computation 3.2: with a return of $0.10\times\text{size}$ and an impact cost of $0.01\times\text{size}^{1.5}$, the optimal size is 44.444444 and the net there is 1.481481.

size	net
10.0000	+0.683772
44.4444	+1.481481
100.0000	+0.000000
200.0000	−8.284271

Observation 3.3

this is the core: powers of 1 and 1.5 put the optimum strictly inside.The return has power 1 and the cost power 1.5, a difference of only 0.5, and yet making Q large enough always lets the cost overtake——a small difference in powers does not mean the absence of an optimum.

Observation 3.4

the shape of the turn is set by the powers, not the coefficients.The net returns to zero at exactly 2.250000 times the optimum, and that ratio moves neither with α nor with β——the number $(3/2)^2$ comes out of the difference of powers alone.

Observation 3.5

so capacity is a property of the strategy.The optimal size is the square of the ratio of α to β, so a better strategy can be run larger, but squared, which makes it swing widely——capacity is a quantity that ought to be reported separately from performance.

4. The Separator

Observation 4.1

the separator is the power with which cost grows with size.At a power of 1 the difference stays proportional, no optimum exists and bigger is better. The moment the power exceeds 1, an interior optimum appears——the boundary sits at the single point 1, and only crossing it matters.

Observation 4.2

beside Paper 335.Paper 335 showed that making a round trip changes the exponent——there the exponent of one quantity was changed by an operation; here two quantities of different exponents compete. Changing an exponent and having exponents compete are different types.

Observation 4.3

what is not said.No particular fund and no execution method is treated, and the impact exponent is representative——what is shown is that a power above 1 forces an optimum, and that its shape follows from the difference of powers.

5. Roll Call and Closing

1. **annual cost 0.252000** (Computation 2.1): **turnover alone.**
2. **gross 2.680000 demanded** (Computation 2.2): **for a net of 1.0.**
3. **rows differ by 0.500000** (Observation 2.3): **added flat.**
4. **31.622777 at tenfold** (Computation 3.1): **faster than size.**
5. **1000.000000 at a hundredfold** (Computation 3.1): **the 1.5 power.**
6. **optimal size 44.444444** (Computation 3.2): **the core.**
7. **net there 1.481481** (Computation 3.2): **the height of the hill.**
8. **−8.284271 at size 200** (Computation 3.2): **overshooting loses.**
9. **zero at 2.250000 times** (Observation 3.4): **fixed by the powers.**
10. **the power of the cost** (Observation 4.1): **the separator; Paper 335.**

Closing: **Return grows with size and cost grows with size to the power 1.5.At 5 bp each way and 252 turnovers a year the annual cost is 0.252000, and a net Sharpe ratio of 1.0 at a volatility of 15 % demands 2.680000 gross——the cost is added flat whatever the target, so reducing turnover sometimes helps more than finding a better strategy.Increasing size increases cost faster still: with impact going as the square root, a hundredfold size gives 1000.000000 times the total cost——a difference of only 0.5 between the powers forces an optimal size of 44.444444 with a net of 1.481481, and at size 200 the net is −8.284271.The net returns to zero at exactly 2.250000 times the optimum, a ratio that moves with neither coefficient——the shape of the turn is set by the difference of powers, not by the coefficients.The separator is the power with which cost grows with size.Placed among the earlier papers——in Paper 335 an operation changed the exponent of one quantity; here two exponents compete.To be honest——no particular fund and no execution is treated, and the impact exponent is representative.**

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