Colour Mixing Is Not One Operation: One Side Adds, the Other Multiplies Transmittances

— two filters of transmittance 0.5 multiply to 0.250000, while averaging gives 0.500000, an error of 2.000000

— the separator is whether the mixing happens on intensity or on transmittance — [Paper 719]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

One phrase, colour mixing, names two different operations. Mixing light adds; stacking paints or filters multiplies.The error from confusing them is counted, and the coordinate in which multiplication becomes addition is shown.No new theorem or law is claimed. First, stacked filters multiply——two sheets of transmittance 0.5 give 0.250000, three give 0.125000, four give 0.062500 (Section 2). **Second, and this is the core. The error from treating it as addition widens with the count——2.000000 at two sheets and 8.000000 at four** (Section 2). **Third, with unequal sheets the product falls below even the smaller one——0.8 with 0.3 gives 0.240000, under the average 0.550000 and under 0.3 itself** (Section 2). **Fourth, in optical density the multiplication becomes an addition——rising by exactly 0.301030 per sheet, so subtractive mixing is not nonlinear but linear in another coordinate** (Section 3). **Fifth, that 0.301030 is its third appearance in this corpus——the digits gained per bisection step in Paper 288 and the frequency of a leading 1 in Paper 612, all three being the share of one decade occupied by a factor of two, agreeing to within 10^{-12}** (Section 3). **Sixth, the separator is whether the mixing happens on intensity or on transmittance——under one word, the place where the addition occurs differs** (Section 4). **Keywords**: additive mixing, subtractive mixing, transmittance, optical density, which quantity is added, shared root.

Scope of this paper (scope note): **No new theorem or law is claimed——the distinction between additive and subtractive mixing, the Beer-Lambert law and the additivity of optical density are all standard (Bouguer 1729, Hunt and Pointer 2011).Vision science is not built——colour-matching functions, adaptation and individual differences in colour vision are not treated.No real material is claimed——spectral properties of inks and pigments are not treated; the transmittances are representative values at a single fixed wavelength.Halftone mixing is not treated——mixing by juxtaposition, as in halftone dots, is not treated; only stacking is.Colour from anything but absorption is not treated ——fluorescence, scattering and interference are not treated.Relation to earlier papers**: **Paper 323 showed that the "three" in three primaries is not a property of light, separating what comes from linearity from what comes from convexity——there the number of primaries was the subject, and the treatment lived on one linear map. Here that linearity is shown to hold only for additive mixing. The linear side, and the side that becomes linear only after taking a logarithm.Papers 288 and 612 are also cited——because 0.301030 appears for the third time, named as a shared root in the sense of Paper 300.**What is added is **showing the error to widen to 8.000000 with the count**, **showing the product to fall below the smaller factor**, **putting the spacing in optical density at 0.301030**, **confirming that value to be the corpus's third**, and **placing the separator on which quantity the addition acts upon**.

1. Notation and the Question

Setting: **superposing light adds intensities. Stacking objects multiplies transmittances.**

$$I_{\text{additive}} = \sum_i I_i, \qquad T_{\text{subtractive}} = \prod_i T_i, \qquad A = -\log_{10} T$$

T is transmittance and A optical density. A single wavelength is fixed throughout, with transmittances taken as representative values.

Question: **does the single phrase "colour mixing" name a single operation?**

2. The Error from Confusing Them

Computation 2.1: **for stacked filters of transmittance 0.5, the product and the average were set side by side.**

sheets	product	average	error
1	0.500000	0.500000	1.000000
2	0.250000	0.500000	2.000000
3	0.125000	0.500000	4.000000
4	0.062500	0.500000	8.000000

> **Observation 2.2**
>
> **this is the core: the error is not constant but widens with the count.2.000000 at two sheets, 8.000000 at four——checking one sheet and declaring it close enough gets worse with every sheet added.**

Computation 2.3: **two sheets of unequal transmittance were stacked.**

quantity	value
product of 0.8 and 0.3	0.240000
average of 0.8 and 0.3	0.550000
the smaller of the two	0.300000

> **Observation 2.4**
>
> **the product is below not only the average but the smaller factor.0.240000 falls under 0.300000——"mixing lands you in between" is a property of addition, and multiplication does not have it.**

> **Observation 2.5**
>
> **the direction is opposite too.Adding all three primaries of light tends to white; stacking all three primaries of colourants tends to black——under the one phrase "three primaries", the destination lies on opposite sides.**

3. The Coordinate in Which Multiplication Becomes Addition

Computation 3.1: the transmittances were converted to optical density.

sheets	transmittance	optical density
1	0.500000	0.301030
2	0.250000	0.602060
3	0.125000	0.903090
4	0.062500	1.204120

> **Observation 3.2**
>
> in optical density the rise is exactly 0.301030 per sheet.The density at four sheets is exactly 4.000000 times that at one——so subtractive mixing is not “nonlinear” but linear in another coordinate.

> **Observation 3.3**
>
> put otherwise, both sides add.Additive mixing adds on intensity; subtractive mixing adds on density——what differs is not the operation but which quantity it acts upon.

> **Observation 3.4**
>
> this 0.301030 is the corpus’s third.In Paper 288 it appeared as the digits gained per bisection step, in Paper 612 as the frequency of a leading 1——derived from three separate formulas the three agree to within 10^{-12}, and all three are the share of one decade occupied by a factor of two.

> **Observation 3.5**
>
> this may be called a shared root in the sense of Paper 300.It is not that the numbers merely coincide: the source is the same, and the source lies on the side of the factor two——built from a factor of three it is off by 0.176091, so the agreement comes from the number two.

4. The Separator

> **Observation 4.1**
>
> **the separator is whether the mixing happens on intensity or on transmittance.Under one word, the place where the addition occurs differs——calling both of them addition is legitimate only once the coordinate is named.**

> **Observation 4.2**
>
> **beside Paper 323.Paper 323 showed that the “three” in three primaries is not a property of light, separating linearity from convexity——there the count was the subject and the treatment lived on one linear map. Here that linearity is shown to hold only for additive mixing: the linear side, and the side that becomes linear only after a logarithm.**

> **Observation 4.3**
>
> **what is not said.Colour-matching functions, adaptation, real ink spectra and mixing by juxtaposition are not treated, and a single wavelength is fixed——what is shown is that one phrase names two operations, and the coordinate that joins them.**

5. Roll Call and Closing

1. **0.250000 at two sheets** (Computation 2.1): **it multiplies.**
2. **0.062500 at four** (Computation 2.1): **further down.**
3. **error 2.000000 at two** (Observation 2.2): **the core.**
4. **8.000000 at four** (Observation 2.2): **it widens with the count.**
5. **0.8 times 0.3 is 0.240000** (Computation 2.3): **below the smaller.**
6. **the average is 0.550000** (Computation 2.3): **for comparison.**
7. **0.301030 per sheet** (Computation 3.1): **evenly spaced.**
8. **1.204120 at four** (Computation 3.1): **exactly four times.**
9. **a factor of three is off by 0.176091** (Observation 3.5): **the source is two.**

10. **which quantity is added** (Observation 4.1): **the separator; Paper 323, Paper 288, Paper 612.**

Closing: **Colour mixing is not one operation: one side adds, the other multiplies transmittances.Two filters of transmittance 0.5 multiply to 0.250000, three to 0.125000, four to 0.062500, while averaging always gives 0.500000——the error is not constant but widens with the count, 2.000000 at two sheets and 8.000000 at four, so checking one sheet and calling it close enough gets worse with every sheet.Unequal sheets behave the same way: the product of 0.8 and 0.3 is 0.240000, under the average 0.550000 and under the smaller factor 0.300000——"mixing lands you in between" belongs to addition and multiplication does not have it.Yet in optical density the multiplication becomes an addition, rising by exactly 0.301030 per sheet to 1.204120 at four——so subtractive mixing is not nonlinear but linear in another coordinate, and what differs is not the operation but which quantity it acts upon.And this 0.301030 is the corpus's third——the digits gained per bisection step in Paper 288 and the frequency of a leading 1 in Paper 612; derived from three separate formulas the three agree to within 10^{-12}, all being the share of one decade occupied by a factor of two, and a factor of three is off by 0.176091.The separator is whether the mixing happens on intensity or on transmittance.Placed among the earlier papers——Paper 323 discussed the count of primaries on one linear map; here the reach of that linearity is cut.To be honest——colour-matching functions, real inks and juxtaposed mixing are not treated, and a single wavelength is fixed.**

References

1. Bouguer, P. (1729). *Essai d'optique sur la gradation de la lumiere*. Claude Jombert.
2. Hunt, R. W. G., & Pointer, M. R. (2011). *Measuring Colour*. Wiley.
3. Yamagishi, Y. (2026). *"Which Is Fastest" Has Three Answers, and a Different Method Wins Each* [Paper 288]. Zenodo. 10.5281/zenodo.22171328
4. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
5. Yamagishi, Y. (2026). *The "Three" in Three Primaries Is Not a Property of Light* [Paper 323]. Zenodo. 10.5281/zenodo.22209319
6. Yamagishi, Y. (2026). *The Leading Digit Is 1 Three Times in Ten, but Only for Data Spread Across Many Orders of Magnitude* [Paper 612]. Zenodo. 10.5281/zenodo.23014476

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)