

Reframing the Cosmological Constant

— Λ as an Integration Constant, Not a Vacuum Energy —

Unimodular Gravity and the $+C$ Duality of Paper 62

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— *From thinking to syncing.* —

Positioning of this paper

This paper offers a **structural interpretation**. It does *not* solve the cosmological constant problem and does not predict the value of Λ . It reframes the problem: in unimodular gravity, Λ is an *integration constant* conjugate to the spacetime **4**-volume, not a vacuum energy to be computed—so the "**120** orders of magnitude" compares two quantities that are not the same object. This develops the $+C/\Lambda$ duality of Paper 62 into a stance on the problem. The reframing removes the radiative fine-tuning but converts the smallness of Λ into a question of boundary conditions; the disciplined success it holds up is Sorkin's fluctuation prediction $\Lambda \sim 1/\sqrt{V_4}$, made before observation. No claim contradicting established theory is made.

ABSTRACT

The cosmological constant problem is usually stated as a fine-tuning: the QFT vacuum energy density exceeds the observed Λ by up to $\sim 10^{120}$, so a bare constant must cancel it to that precision. This presupposes that vacuum energy *sources* Λ . This paper develops the reading of Paper 62— Λ canonically conjugate to the **4**-volume V_4 , dual to the volume-baseline integration constant $+C$ of Paper 12—into a reframing via **unimodular gravity**. There the metric determinant is fixed and the field equations are trace-free; a constant vacuum energy is pure trace and so *does not source* them. The effective cosmological constant re-emerges as a *constant of integration*, conjugate to V_4 (Henneaux–Teitelboim 1989). Thus Λ is boundary data, not a computed vacuum energy, and the "**120** orders" is a **category error**. We are explicit about the limit: unimodular gravity removes the fine-tuning but does not *predict* Λ ; the smallness becomes a boundary-condition question. We hold up, as the model of a disciplined structural prediction, Sorkin's causal-set argument that $\Lambda \sim \pm 1/\sqrt{V_4} \sim H_0^2$ —from the same conjugacy—which anticipated the observed order before 1998.

1. Introduction

The cosmological constant problem is the largest numerical mismatch in physics. Its standard statement is a fine-tuning: QFT assigns the vacuum a large energy density ρ_{vac} that gravitates as $\Lambda_{\text{eff}} = \Lambda_{\text{bare}} + 8\pi G \rho_{\text{vac}}$; since ρ_{vac} exceeds the observed value by up to $\sim 10^{120}$, Λ_{bare} must be tuned to cancel it to **120** digits.

This contains an assumption: that vacuum energy *sources* Λ . Paper 62 already suggested a different status: reading Λ , as in unimodular gravity, as *conjugate to the 4-volume* V_4 (Henneaux–Teitelboim), dual to the volume-baseline integration constant $+C$ of $\int 4\pi r^2 dr = \frac{4}{3}\pi r^3 + C$ (Paper 12). This paper develops that into a stance: if Λ is an integration constant, computing it from ρ_{vac} is a category error.

What this paper does not do. It does not solve the problem and does not predict Λ . It relocates the question honestly and points to where a genuine structural prediction has been made (Sorkin).

2. The Problem, as Usually Posed

Standard general relativity treats Λ as a coupling in the action:

$$S = \frac{1}{16\pi G} \int (R - 2\Lambda) \sqrt{-g} d^4x + S_{\text{matter}}$$

into which any constant shift of the matter Lagrangian—in particular a vacuum energy ρ_{vac} —feeds directly, since $\int \rho_{\text{vac}} \sqrt{-g} d^4x$ has exactly the form of a cosmological term. Hence $\Lambda_{\text{eff}} = \Lambda_{\text{bare}} + 8\pi G \rho_{\text{vac}}$, requiring a ~ 120 -order cancellation. The problem is real *given* the premise that vacuum energy gravitates as a cosmological term.

3. Unimodular Gravity: Λ as an Integration Constant

In unimodular gravity the metric determinant is fixed, $\sqrt{-g} = \varepsilon_0$, so the metric loses one degree of freedom. Varying under this constraint yields not the full Einstein equations but their *trace-free* part:

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = 8\pi G \left(T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T \right)$$

Proposition 3.1 (Vacuum energy decouples; Λ is an integration constant)

A constant vacuum energy contributes $T_{\mu\nu}^{\text{vac}} = -\rho_{\text{vac}} g_{\mu\nu}$, pure trace: $T_{\mu\nu}^{\text{vac}} - \frac{1}{4} g_{\mu\nu} T^{\text{vac}} = 0$. Hence ρ_{vac} does *not* source the trace-free equations. Taking the divergence and using the Bianchi identity re-introduces a constant of integration Λ , entering the recovered Einstein equations exactly as a cosmological term. In the diffeomorphism-invariant formulation of Henneaux–Teitelboim (1989), this Λ is canonically conjugate to V_4 . Therefore Λ is fixed by boundary/initial data, not computed from ρ_{vac} .

Consequently, in unimodular gravity a shift $\rho_{\text{vac}} \rightarrow \rho_{\text{vac}} + \text{const}$ is **unobservable**—it does not change the equations and is absorbed into the integration constant. The huge QFT vacuum energy is decoupled from the physical Λ .

4. The $+C$ Reading (Paper 62)

Corollary 4.1 (Λ is a constant of integration, like $+C$)

Proposition 3.1 is the $+C/\Lambda$ duality of Paper 62 made into a stance. In $\int 4\pi r^2 dr = \frac{4}{3}\pi r^3 + C$ (Paper 12), the baseline $+C$ is an integration constant of the volume—undetermined by the integrand. Paper 62 identified Λ , conjugate to V_4 , as its dual on the volume degree of freedom. Unimodular gravity says the same dynamically: Λ is a constant of integration on the volume. **One no more computes Λ from the vacuum energy than one computes the $+C$ of an indefinite integral from its integrand.**

5. The Category Error

Observation 5.1 (The "120 orders" is a category error)

The number $\sim 10^{120}$ measures the ratio of the QFT vacuum energy density to the observed Λ , *under the premise that the former sources the latter*. By Proposition 3.1, in unimodular gravity that premise is false: ρ_{vac} does not source Λ , a boundary datum. The "120 orders" thus compares two quantities of different kind—a computed vacuum energy and a constant of integration—that the theory does not identify. It is not the fine-tuning of a single number but a mismatch of categories.

This does not make the number vanish; it relocates its meaning. The mismatch is real evidence that "vacuum energy is Λ " is the wrong bookkeeping. Reframing is not solving, but it says **where not to look**.

6. What This Does and Does Not Achieve

Honesty requires separating two problems traditionally bundled:

- The *radiative/fine-tuning* problem—why the bare constant must cancel a huge vacuum energy—is **removed**, because vacuum energy decouples (Proposition 3.1).
- The *value* problem—why the integration constant is small—is **not** solved. It becomes a question of boundary/initial conditions or of a dynamical/statistical mechanism.

On the value problem we point to a genuine, disciplined structural prediction. Sorkin's causal-set argument uses the same Λ – V_4 conjugacy: if Λ and V_4 are conjugate, fluctuations give $\Delta\Lambda \sim 1/\sqrt{V_4}$ (Planck units), so $\Lambda \sim H_0^2$ at present, of the observed order. **This was made before the 1998 supernova observations and anticipated the right magnitude.** It is the model of what a structural reframing can earn: not a re-labelling, but a prediction that differs from the standard expectation and is confirmed—exactly the criterion for promoting a structural reading to a physical one.

7. Scope of the Claims

What is established

1. In unimodular gravity a constant vacuum energy does not source the trace-free equations, and Λ re-emerges as an integration constant conjugate to V_4 (Proposition 3.1).
2. This realises the $+C/\Lambda$ duality of Paper 62 dynamically (Corollary 4.1); computing Λ from ρ_{vac} is a category error (Observation 5.1).

What is not established / Open problem

- A solution of the cosmological constant problem. The value of Λ is not predicted; the fine-tuning is removed but the smallness becomes a boundary-condition question.
- That unimodular gravity is the unique or complete framework. It is one established, non-string reframing, its interpretation discussed in the literature (e.g. its classical equivalence to GR with an added constant).
- Any new physics. The unimodular field equations and the Sorkin estimate are cited, not derived here.
- *Open*: is there a principle fixing the integration constant—a boundary condition, or a Sorkin-type fluctuation law sharpened to a testable time-dependence of Λ —that would turn the reframing into a prediction? The Λ – V_4 conjugacy (Papers 62, 12) is the natural place to seek it.

8. Conclusion

The cosmological constant problem is usually posed as the fine-tuning of a bare constant against a huge vacuum energy. Unimodular gravity changes the status of Λ : with the metric determinant fixed, the field equations are trace-free, a constant vacuum energy does not source them, and Λ re-emerges as a constant of integration conjugate to the 4-volume—exactly the $+C/\Lambda$ duality of Paper 62. So Λ is boundary data, not a computed vacuum energy, and the "120 orders" is a category error. This removes the fine-tuning without predicting the value; the smallness of Λ becomes a question of boundary conditions, where Sorkin's estimate $\Lambda \sim 1/\sqrt{V_4} \sim H_0^2$, drawn from the same conjugacy and made before observation, shows what a disciplined structural prediction looks like. **We reframe, and we say honestly what remains.**

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