Under the One Word Dyeing Live an Expression with a Ceiling and One Without, Indistinguishable When Dilute

— In the site-filling kind the number of sites is a ceiling, so even at a concentration of **1000** the occupancy is **0.999500** and never exceeds one — the separator is whether the dye's destination is a finite set of sites or a volume — [Paper 721]

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— From thinking to syncing. —

Abstract

A dye entering a fibre and settling there is called dyeing. Beneath that word lie two mechanisms: one that saturates and one that takes in as much as offered.That the two diverge at high concentration and coincide at low is shown in numbers.No new theorem or law is claimed. First, the site-filling kind has a ceiling——even at a concentration of 1000 the occupancy is 0.999500 and never exceeds one (Section 2). **Second, in that kind the gain shrinks——a tenfold rise in concentration gives 1.428571 at concentration 1 and 1.044776 at 10** (Section 2). **Third, the volume kind has no ceiling——a tenfold rise always gives exactly 10.000000, reaching 20000.000000 at a concentration of 1000** (Section 3). **Fourth, and this is the core. At low concentration the two cannot be told apart——between 0.001 and 0.01 even the site kind gives 9.823529, close to proportionality** (Section 3). **Fifth, so the presence of a ceiling cannot be settled on the dilute side——telling them apart requires raising the concentration until the ceiling is approached** (Section 3). **Sixth, the separator is whether the destination is a finite set of sites or a volume——sites can be counted; a volume cannot** (Section 4). **Keywords**: dyeing, adsorption isotherm, partition law, presence of a ceiling, dilute limit, one name over two forms.

Scope of this paper (scope note): **No new theorem or law is claimed——the Langmuir isotherm, the Nernst partition law, and the distinction between these two kinds in dyeing are all standard (Langmuir 1918, Atkins and de Paula 2010).No real combination is claimed——values for particular dye-fibre pairs are not treated; the equilibrium constant and the partition coefficient are representative values assumed here.Rates are not treated——the rate of dyeing, and whether it is diffusion- or reaction-limited, are not entered; only equilibrium is considered.Conditions are not discussed——temperature dependence, pH and electrolyte effects are not treated.The process is not designed——liquor ratio and dyeing procedure are not treated.Relation to earlier papers**: **Paper 284 showed that the Langmuir isotherm and the Fermi distribution are one and the same expression——there the subject was different names sharing one form, and its scope note stated that the mechanism of adsorption would not be discussed and the distinction between chemical and physical adsorption not treated at all. Here the direction is reversed: one name taking two different forms. Same form under different names, and same name over different forms.**What is added is **putting the site kind's ceiling at $\mathbf{0.999500}$**, **showing the gain to shrink from $\mathbf{1.428571}$ to $\mathbf{1.044776}$**, **showing the volume kind to reach $\mathbf{20000.000000}$**, **showing the two to coincide at $\mathbf{9.823529}$ on the dilute side**, and **placing the separator on sites against volume**.

1. Notation and the Question

Setting: **a dye moving to the fibre side has two kinds of destination: a finite set of sites, or a volume it dissolves into.**

$$\theta = \frac{Kc}{1 + Kc}, \qquad c_{\text{fibre}} = K_p\, c$$

θ is the site occupancy and c the bath concentration. Below, $K = 2$ and $K_p = 20$ are taken as representative values.

Question: **does the single word "dyeing" name a single mechanism?**

2. The Site-Filling Kind Has a Ceiling

Computation 2.1: **for the site kind, the occupancy was derived against concentration.**

concentration	occupancy
0.1	0.166667
1	0.666667
10	0.952381
100	0.995025
1000	0.999500

> **Observation 2.2**
>
> no concentration takes it above one.At a concentration of 1000 it is 0.999500——the number of sites is the ceiling, and once they are filled the remaining dye stays in the bath.

Computation 2.3: the gain from a tenfold rise in concentration was set side by side for the two kinds.

concentration	site kind	volume kind
0.1 → 1	4.000000	10.000000
1 → 10	1.428571	10.000000
10 → 100	1.044776	10.000000

> **Observation 2.4**
>
> in the site kind the gain itself depends on concentration.It shrinks from 1.428571 to 1.044776——the range over which “twice the dye gives twice the depth” holds is itself set by the concentration.

3. The Volume Kind Has No Ceiling, and the Dilute Side Hides the Difference

Computation 3.1: for the volume kind, the concentration in the fibre was derived against the bath concentration.

bath concentration	concentration in the fibre
0.1	2.000000
1	20.000000
10	200.000000
100	2000.000000
1000	20000.000000

Observation 3.2

this one has no ceiling.A tenfold rise always gives exactly 10.000000, reaching 20000.000000 at a concentration of 1000——the destination is a volume rather than a set of sites, so no countable upper bound exists.

Observation 3.3

this is the core: at low concentration the two cannot be told apart.Between 0.001 and 0.01 even the site kind gives 9.823529——the gap from 10.000000 is small, so measuring only the dilute side and reading proportionality misses a kind that has a ceiling.

Observation 3.4

telling them apart therefore requires raising the concentration.Raise it until the ceiling is approached and see whether proportionality breaks——a good fit on the dilute side says nothing about which kind it is.

Observation 3.5

the difficulty comes from the form of the expression.The site kind is asymptotically proportional in the dilute limit, so the two kinds must coincide there——this is structural, not accidental.

4. The Separator

Observation 4.1

the separator is whether the destination is a finite set of sites or a volume.Sites can be counted; a volume cannot——under the one word “dyeing”, whether an upper bound exists changes.

Observation 4.2

beside Paper 284.Paper 284 showed the Langmuir isotherm and the Fermi distribution to be one expression——there different names shared one form. Here the direction is reversed: one name takes two forms. Two ways for name and form to diverge.

Observation 4.3

what is not said.Real combinations, rates, temperature, pH and process design are not treated, and both constants are representative——what is shown is that one word contains two kinds, and that the dilute side does not reveal it.

5. Roll Call and Closing

1. **0.666667 at concentration 1** (Computation 2.1): **the site kind.**
2. **0.952381 at 10** (Computation 2.1): **the ceiling is near.**
3. **0.999500 at 1000** (Computation 2.1): **never above one.**
4. **gain 1.428571** (Computation 2.3): **concentration 1 → 10.**
5. **then 1.044776** (Computation 2.3): **it shrinks.**
6. **volume kind 10.000000** (Computation 2.3): **always the same.**
7. **20000.000000 at 1000** (Computation 3.1): **no ceiling.**
8. **9.823529 on the dilute side** (Observation 3.3): **the core.**
9. **they coincide structurally** (Observation 3.5): **not accidental.**
10. **sites or volume** (Observation 4.1): **the separator; Paper 284.**

Closing: **Under the one word dyeing live two expressions, one with a ceiling and one without, and at low concentration they cannot be told apart.In the site-filling kind the number of sites is the ceiling, so even at a concentration of 1000 the occupancy is 0.999500 and never exceeds one, while the gain from a tenfold rise shrinks from 1.428571 to 1.044776——the range over which "twice the dye gives twice the depth" holds is itself set by the concentration.The kind that dissolves into a volume has no ceiling, so a tenfold rise always gives exactly 10.000000, reaching 20000.000000 at a concentration of 1000——the destination is a volume rather than a set of sites, so no countable upper bound exists.Yet between 0.001 and 0.01 even the site kind gives 9.823529, close to 10.000000——so measuring only the dilute side and reading proportionality misses a kind that has a ceiling.Telling them apart requires raising the concentration until the ceiling is approached and seeing whether proportionality breaks——a good fit on the dilute side says nothing about which kind it is.And the difficulty is structural rather than accidental: the site kind is asymptotically proportional in the dilute limit, so the two must coincide there.The separator is whether the destination is a finite set of sites or a volume.Placed among the earlier papers——in Paper 284 different names shared one form; here one name takes two forms.To be honest——real combinations, rates and conditions are not treated, and both constants are representative.**

References

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