Because What Is Being Carried Is the Fuel Itself, the First Stretch and the Last Stretch of One Flight Cost Different Amounts

— in the Breguet equation doubling the dimensionless range x from 1 to 2 takes 3.718282 times the fuel, and at $x = 1$ the first kilometre costs 2.718282 times the last — the separator is whether the distance comes from stored height or from stored fuel — [Paper 723]

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— From thinking to syncing. —

Abstract

Load more fuel and you go further. But the fuel loaded must itself be carried, so distance and fuel are not proportional.That different stretches of one flight cost different amounts is shown in numbers.No new theorem or law is claimed. First, the fuel needed is exponential in distance——against the dimensionless distance x it is 1.718282 at $x = 1$, 6.389056 at 2 and 19.085537 at 3 (Section 2). **Second, so doubling the distance takes more than double the fuel——taking x from 1 to 2 takes 3.718282 times the fuel** (Section 2). **Third, and this is the core. Different stretches of one flight cost differently——fuel flow being proportional to weight, at $x = 1$ the first kilometre costs 2.718282 times the last** (Section 3). **Fourth, the comparison is unpowered gliding——there the distance is the product of height and lift-to-drag ratio, staying at 15000.000000 m even when the weight is varied by a factor of 125** (Section 3). **Fifth, so weight enters in two different ways——in gliding it drops out of the expression; under power it remains inside a logarithm, and what is carried is the weight itself** (Section 3). **Sixth, the separator is whether the distance comes from stored height or from stored fuel——in the first the weight disappears; in the second it rebounds on itself** (Section 4). **Keywords**: Breguet equation, range, fuel weight fraction, glide distance, how weight enters, cost per stretch.

Scope of this paper (scope note): **No new theorem or law is claimed——the Breguet range equation, the setting of glide distance by lift-to-drag ratio, and the proportionality of fuel flow to weight are all standard (Breguet 1923, Anderson 1999).No real aircraft is claimed——specific fuel consumption and lift-to-drag ratio are not treated separately but folded into one constant K.Only cruise is considered——fuel for climb, descent and holding is not included.Lift-to-drag ratio is held constant——its variation during flight is not treated.No diagram is drawn——the trade between payload and range is not treated.$x = 1$ is a normalisation——the 2.718282 appearing there follows from setting the scale of x to one; it is not a measured value.Relation to earlier papers**: **Paper 590 showed that a clip on a paper aeroplane does not shorten how far it glides——there the weight dropped out of the distance and adding weight did not shorten it. Here the weight not only remains inside a logarithm: what is carried is the fuel itself, so the weight rebounds on itself. The side where weight disappears, and the side where it returns.**What is added is **tabulating the fuel needed as an exponential in distance**, **putting the cost of doubling at 3.718282**, **deriving the difference in cost between stretches of one flight**, **confirming over a factor of 125 that weight drops out of gliding**, and **placing the separator on stored height against stored fuel**.

1. Notation and the Question

Setting: **the range in cruise is proportional to the logarithm of the ratio of take-off to landing weight.**

$$R = K \ln \frac{W_i}{W_f}, \qquad x = \frac{R}{K}, \qquad \frac{W_{\text{fuel}}}{W_f} = e^x - 1$$

K folds together speed, specific fuel consumption and lift-to-drag ratio, and has the dimension of length. W_f is payload plus empty weight and x the dimensionless range.

Question: **are distance and fuel proportional?**

2. The Fuel Needed Is Exponential in Distance

Computation 2.1: **the weight ratio and the fuel needed were derived for each dimensionless distance.**

x	weight ratio	fuel against W_f
0.5	1.648721	0.648721
1.0	2.718282	1.718282
1.5	4.481689	3.481689
2.0	7.389056	6.389056
3.0	20.085537	19.085537

Observation 2.2

doubling the distance takes more than double the fuel.Taking x from 1 to 2 moves the fuel from 1.718282 to 6.389056, a factor of 3.718282——because the fuel loaded must itself be carried.

Observation 2.3

yet the fuel fraction never reaches one.Even at $x = 3$ the fuel is 0.950213 of the take-off weight——the expression places no upper bound on distance; the bound comes from the structure and from what one wants to carry.

3. Within One Flight, Stretches Cost Differently

Observation 3.1

this is the core: fuel flow is proportional to weight.So a kilometre is dear while the aircraft is heavy and cheap once it is light——at $x = 1$ the first kilometre costs 2.718282 times the last, and at $x = 2$, 7.389056 times.

Observation 3.2

that factor is the weight ratio itself.The ratio of weights at the start and end of the flight is exactly the ratio of the cost of a kilometre——within one distance, stretches of different price stand side by side.

Computation 3.3: **unpowered gliding was set up as a comparison.**

quantity	value
glide from **1000** m at a lift-to-drag ratio of **15**	**15000.000000** m
spread in distance over a factor of **125** in weight	**0**

> **Observation 3.4**
>
> **in gliding the weight drops out of the expression.The distance is height times lift-to-drag ratio, staying at 15000.000000 m over a factor of 125 in weight——a heavier craft descends faster, but travels faster by the same factor, so the distance is unchanged.**

> **Observation 3.5**
>
> **so weight enters in two ways.Fly on stored height and the weight disappears; fly on stored fuel and it remains inside a logarithm——and in the second case what is carried is the fuel itself, so the weight rebounds on itself.**

4. The Separator

> **Observation 4.1**
>
> **the separator is whether the distance comes from stored height or from stored fuel.In the first the weight disappears; in the second it returns——“heavy things do not fly far” changes its truth value with the case.**

> **Observation 4.2**
>
> **beside Paper 590.Paper 590 showed that a clip on a paper aeroplane does not shorten how far it glides——there the weight disappeared. Here it remains, and what is carried is the weight itself. The side where weight disappears, and the side where it returns.**

Observation 4.3

what is not said.Real aircraft, climb and descent, and variation in lift-to-drag ratio are not treated, and $x = 1$ is a normalisation rather than a measurement——what is shown follows from the fact that what is carried is the fuel itself.

5. Roll Call and Closing

1. **1.718282 of fuel at $x = 1$** (Computation 2.1): **it is exponential.**
2. **6.389056 at $x = 2$** (Computation 2.1): **doubling is not enough.**
3. **19.085537 at $x = 3$** (Computation 2.1): **it jumps further.**
4. **doubling costs 3.718282** (Observation 2.2): **fuel carries fuel.**
5. **fuel fraction 0.950213** (Observation 2.3): **it never reaches one.**
6. **first kilometre 2.718282 times** (Observation 3.1): **the core.**
7. **7.389056 at $x = 2$** (Observation 3.1): **the weight ratio itself.**
8. **glide of 15000.000000 m** (Computation 3.3): **the comparison.**
9. **spread 0 over a factor of 125** (Observation 3.4): **weight disappears.**
10. **height or fuel** (Observation 4.1): **the separator; Paper 590.**

Closing: **Because what is being carried is the fuel itself, the first stretch and the last stretch of one flight cost different amounts.Against the dimensionless range x, the fuel needed rises exponentially: 0.648721 at $x = 0.5$, 1.718282 at 1, 6.389056 at 2 and 19.085537 at 3——so taking x from 1 to 2 takes 3.718282 times the fuel, and doubling is not enough.And since fuel flow is proportional to weight, at $x = 1$ the first kilometre costs 2.718282 times the last, and at $x = 2$, 7.389056 times——that factor is the ratio of weights at the start and end of the flight, so stretches of different price stand side by side within one distance.The comparison is unpowered gliding——there the distance is height times lift-to-drag ratio, staying at 15000.000000 m over a factor of 125 in weight: a heavier craft descends faster but travels faster by the same factor.So weight enters in two ways——fly on stored height and it disappears; fly on stored fuel and it remains inside a logarithm, rebounding on itself because what is carried is the fuel.The separator is whether the distance comes from stored height or from stored fuel.Placed among the earlier papers——in Paper 590 the weight disappeared; here it returns.To be honest——real aircraft and the climb and descent are not treated, and $x = 1$ is a normalisation rather than a measurement.**

References

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