Climbing Does Not Reduce the Drag; What Altitude Buys Is the Speed at Which the Same Drag Is Produced

— the drag is the weight divided by the lift-to-drag ratio, so across five altitudes it stays at **117647.058824** N, and what altitude buys is speed at the same drag — the separator is whether altitude acts on the magnitude of the force or on the speed at which it is produced — [Paper 724]

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September 2026

— *From thinking to syncing.* —

Abstract

Airliners are said to fly high because thin air means less drag. The drag has not become less.What changes and what does not is separated within one atmospheric table.No new theorem or law is claimed. First, density at 11000 m is under a third of its sea-level value——from 1.225000 kg/m³ to 0.363918 kg/m³, a factor of 3.366147 (Section 2). **Second, so the speed needed for the same lift rises——as the −1/2 power of density, by 1.834706** (Section 2). **Third, and this is the core. The drag does not move——being the weight divided by the lift-to-drag ratio, it stays at 117647.058824 N across five altitudes** (Section 3). **Fourth, so what altitude buys is speed, not force——if fuel consumption is proportional to thrust, the distance grows by the ratio of speeds** (Section 3). **Fifth, the ceiling comes from the speed of sound——set by temperature, it falls to 0.867102 of its sea-level value, so the Mach number rises further still, by 2.115907** (Section 3). **Sixth, the separator is whether altitude acts on the magnitude of the force or on the speed at which it is produced——the first does not move; only the second does** (Section 4). **Keywords**: standard atmosphere, cruise altitude, lift-to-drag ratio, drag, true airspeed, Mach number.

Scope of this paper (scope note): **No new theorem or law is claimed——the standard atmosphere, the lift equation, the setting of drag in level flight by weight and lift-to-drag ratio, and the dependence of the speed of sound on temperature are all standard (Anderson 1999, Houghton and Carpenter 2003).How cruise altitude is actually chosen is not claimed——specific fuel consumption, the altitude characteristics of thrust and the drift of the optimum with weight are not treated.The drag coefficient is held constant——its variation with speed and Mach number is not treated; one lift-to-drag ratio is assumed.Stratospheric structure is not treated——the expression is merely switched at the tropopause.The coffin corner is not located——where the stall and Mach limits meet is pointed to but not computed.Relation to earlier papers**: **Paper 592 showed that a rubber balloon swells as it rises but its ascent speeds up only as the $-1/6$ power of the air density——there the same density of $\mathbf{0.363918}$ kg/m³ at the same 11000 m was used, and against the same density ratio of $\mathbf{3.366147}$ the speed rose by only $\mathbf{1.224209}$. Here the same ratio gives $\mathbf{1.834706}$ at the $-1/2$ power. The $-1/2$ power is exactly the cube of the $-1/6$ power, agreeing to within 10^{-15}. The same quantity under different exponents.**What is added is **deriving the speed ratio from the density**, **showing the drag to be independent of altitude**, **separating the gain as speed rather than force**, **putting the rise in Mach number at $\mathbf{2.115907}$**, and **placing the separator on force against speed**.

1. Notation and the Question

Setting: **in level flight the lift equals the weight, and the drag is the weight divided by the lift-to-drag ratio.**

$$W = \tfrac{1}{2}\rho V^2 S C_L, \qquad D = \frac{W}{L/D}, \qquad a = \sqrt{\gamma R T}$$

a is the speed of sound. Below, a weight of 2.0×10^6 N and a lift-to-drag ratio of 17 are held fixed and only the altitude is varied.

Question: **what exactly is gained by flying high?**

2. Density Falls, So Speed Rises

Computation 2.1: **the standard-atmosphere density and the ratio of speeds needed for the same lift were derived.**

altitude	density	speed needed (sea level as 1)
0 m	1.225000 kg/m³	1.000000
3000 m	0.909122 kg/m³	1.160799
6000 m	0.659697 kg/m³	1.362686
9000 m	0.466348 kg/m³	1.620739
11000 m	0.363918 kg/m³	1.834706

Observation 2.2

the ratio of speeds is the $-1/2$ power of density.A density ratio of 3.366147 gives 1.834706——since lift is the product of density and the square of speed, making up a fall in density takes only its square root in speed.

Observation 2.3

the same density can carry a different exponent.Paper 592 derived a $-1/6$ power against the same density ratio, giving only 1.224209——the $-1/2$ power is exactly the cube of the $-1/6$ power, agreeing to within 10^{-15}: the exponent alone changes the growth.

3. The Drag Does Not Move

Computation 3.1: **at the same weight and lift-to-drag ratio, the drag was derived for each altitude.**

altitude	drag
0 m	117647.058824 N
6000 m	117647.058824 N
11000 m	117647.058824 N

Observation 3.2

this is the core: across five altitudes the spread in drag is 0.The drag is the weight divided by the lift-to-drag ratio, with neither density nor speed in it——thin air lowers the dynamic pressure, but one flies faster by the same amount, and the product returns.

Observation 3.3

so what altitude buys is speed, not force.The thrust is the same, and the distance covered on that thrust grows——if fuel consumption is proportional to thrust, the same fuel carries 1.834706 times as far.

Observation 3.4

“thin air makes it easier” is not an accurate phrasing.What has become easier is not the force——the same force now carries the aircraft faster.

Computation 3.5: **to see the ceiling, temperature and speed of sound were derived for each altitude.**

altitude	temperature	speed of sound
0 m	**288.150000** K	**340.293947** m/s
6000 m	**249.150000** K	**316.428329** m/s
11000 m	**216.650000** K	**295.069458** m/s

Observation 3.6

the ceiling comes from the speed of sound.It falls to 0.867102 of its sea-level value while the speed needed rises by 1.834706——so the Mach number rises by 2.115907, and the two limits close on each other with altitude.

4. The Separator

> **Observation 4.1**
>
> **the separator is whether altitude acts on the magnitude of the force or on the speed at which it is produced.The first does not move; only the second does——without separating which one is meant by “the gain”, it sounds as though the drag had fallen.**

> **Observation 4.2**
>
> **beside Paper 592.Paper 592 showed a balloon’s ascent to speed up only as the $-1/6$ power of the air density——the same atmosphere and the same density, with only the exponent differing. One quantity carrying another exponent.**

> **Observation 4.3**
>
> **what is not said.How cruise altitude is chosen, the altitude characteristics of thrust and the variation of the drag coefficient are not treated, and the coffin corner is not located——what is shown is the line between what moves and what does not.**

5. Roll Call and Closing

1. **sea-level density 1.225000** (Computation 2.1): **the reference.**
2. **0.363918 at 11000 m** (Computation 2.1): **a factor of 3.366147.**
3. **speed needed 1.834706** (Computation 2.1): **the $-1/2$ power.**
4. **1.362686 at 6000 m** (Computation 2.1): **a value in between.**
5. **Paper 592 gives 1.224209** (Observation 2.3): **the $-1/6$ power.**
6. **drag 117647.058824 N** (Computation 3.1): **the core.**
7. **spread 0 across altitudes** (Observation 3.2): **the force does not move.**
8. **speed of sound 0.867102** (Computation 3.5): **set by temperature.**
9. **Mach number 2.115907** (Observation 3.6): **the limits close in.**
10. **force or speed** (Observation 4.1): **the separator; Paper 592.**

Closing: **Climbing does not reduce the drag; what altitude buys is the speed at which the same drag is produced.In the standard atmosphere the density falls from 1.225000 kg/m³ at sea level to 0.363918 kg/m³ at 11000 m, a factor of 3.366147, so the speed needed for the same lift rises as the $-1/2$ power, by 1.834706——since lift is the product of density and the square of speed, making up a fall in density takes only its square root in speed.Yet the drag, being the weight divided by the lift-to-drag ratio, stays at 117647.058824 N across five altitudes, a spread of 0——thin air lowers the dynamic pressure, but one flies faster by the same amount and the product returns.So what altitude buys is speed rather than force: the thrust is unchanged and the distance covered on it grows——"thin air makes it easier" is not accurate, because what has become easier is not the force.The ceiling comes from the speed of sound ——set by temperature, it falls to 0.867102 of its sea-level value while the speed needed rises by 1.834706, so the Mach number rises by 2.115907 and the two limits close on each other with altitude.The separator is whether altitude acts on the magnitude of the force or on the speed at which it is produced.Placed among the earlier papers ——Paper 592 put a $-1/6$ power on the same density and got 1.224209; here the power is $-1/2$.To be honest—— how cruise altitude is chosen and the variation of the drag coefficient are not treated, and the coffin corner is not located.**

References

1. Anderson, J. D. (1999). *Aircraft Performance and Design*. McGraw-Hill.
2. Houghton, E. L., & Carpenter, P. W. (2003). *Aerodynamics for Engineering Students*. Butterworth-Heinemann.
3. Yamagishi, Y. (2026). *A Rubber Balloon Swells as It Rises, but Its Ascent Speeds Up Only as the $-1/6$ Power of the Air Density* [Paper 592]. Zenodo. 10.5281/zenodo.23014392

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