The Take-off Decision Speed Is Not Chosen but Where Two Distances Cross, and That Crossing Is the Minimum

— the distance to stop rises with speed and the distance to continue falls, so exactly one crossing exists, at $V_1 = 74.551289$ m/s and 2315.789474 m — the separator is whether the two distances move the same way with speed or opposite ways — [Paper 725]

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— *From thinking to syncing.* —

Abstract

When an engine fails during take-off, a speed divides stopping from continuing. It is not a value somebody chose but the point where two curves cross.That there is exactly one crossing, and that it is the minimum, is shown in numbers.No new theorem or law is claimed. First, the distance to stop rises with speed——666.666667 m at 40 m/s and 2041.666667 m at 70 m/s (Section 2). **Second, the distance to continue falls with speed——3800.000000 m and 2562.500000 m at the same two points** (Section 2). **Third, so exactly one crossing exists——the difference is monotone and its sign is opposite at the ends, negative at 1 m/s and positive at 80 m/s** (Section 2). **Fourth, the crossing is $V_1 = 74.551289$ m/s at 2315.789474 m——obtained by bisection, with the two distances differing there by under 10^{-6} m** (Section 3). **Fifth, and this is the core. The crossing is the minimum——the length required is the larger of the two, so lowering V_1 by 10 m/s gives 2837.424143 m and raising it gives 2978.716885 m** (Section 3). **Sixth, the separator is whether the two distances move the same way with speed or opposite ways——the same way and there is no crossing, and the notion of a decision speed does not stand** (Section 4). **Keywords**: accelerate-stop distance, take-off continued distance, decision speed, uniqueness of the crossing, the point that minimises, direction of the two distances.

Scope of this paper (scope note): **No new theorem or law is claimed——accelerate-stop distance, take-off continued distance and the balanced field length are all standard (Anderson 1999, Torenbeek 1982).No real V_1 is claimed——the accelerations, lift-off speed and air distance are representative values assumed here, and the regulatory definition, including that V_1 has a range of speeds and how certification proceeds, is not treated.Runway condition is not treated ——wetness, slope and wind are not treated.Delay is not included——the time between recognising and acting is not included; including it would lengthen only the stopping side.The air distance is not derived——the 400.0 m after lift-off is assumed and not derived from an obstacle height.Accelerations are held constant——the variation of thrust and drag with speed is not treated.Relation to earlier papers**: **Paper 723 showed that because what is carried is the fuel itself, stretches of one flight cost different amounts——there the distance was set by the logarithm of the fuel, a one-directional trade that grows when one adds. Here the distance is set by the larger of two opposing requirements, so it grows whichever way one moves. A one-directional trade, and a minimum squeezed from both sides.**What is added is **showing numerically that the two distances move opposite ways**, **supporting the uniqueness of the crossing by the signs at the ends**, **putting the crossing at 74.551289 m/s**, **confirming on both sides that it is the minimum**, and **placing the separator on the direction of the two distances**.

1. Notation and the Question

Setting: **suppose an engine fails at speed v. To stop, add the braking distance to the distance already covered; to continue, carry the remaining acceleration on the remaining engines to lift-off and add the air distance.**

$$d_{\text{stop}}(v) = \frac{v^2}{2a} + \frac{v^2}{2a_b}, \qquad d_{\text{go}}(v) = \frac{v^2}{2a} + \frac{V_{lo}^2 - v^2}{2a_1} + d_{\text{air}}$$

a is the acceleration on all engines, a_1 on one out, a_b the braking deceleration. Take $a = 2.0$, $a_1 = 0.8$, $a_b = 3.0$ m/s², $V_{lo} = 80.0$ m/s and $d_{\text{air}} = 400.0$ m.

Question: **where does the speed dividing stopping from continuing come from?**

2. The Two Distances Move Opposite Ways

Computation 2.1: **the two distances were derived for each speed.**

speed	distance to stop	distance to continue	difference
40 m/s	666.666667 m	3800.000000 m	−3133.333333 m
50 m/s	1041.666667 m	3462.500000 m	−2420.833333 m
60 m/s	1500.000000 m	3050.000000 m	−1550.000000 m
70 m/s	2041.666667 m	2562.500000 m	−520.833333 m
80 m/s	2666.666667 m	2000.000000 m	+666.666667 m

Observation 2.2

the directions are opposite.The distance to stop rises as the square of speed while the distance to continue falls——running faster before stopping takes more room, while having reached a higher speed leaves a shorter stretch to be carried on the remaining engines.

Observation 2.3

so exactly one crossing exists.The difference is monotone, negative at 1 m/s and positive at 80 m/s——the sign changes once, so the crossing is unique.

Observation 2.4

the crossing rests on the remaining engines accelerating less.Were the one-out acceleration the same, the distance to continue would not depend on speed at all——the notion of a decision speed comes from the single fact that losing an engine costs acceleration.

3. The Crossing Is the Minimum

Computation 3.1: **the crossing was found by bisection and the required length derived for displaced values of V_1.**

V_1	length required (the larger of the two)
64.551289 m/s	2837.424143 m
69.551289 m/s	2585.981809 m
74.551289 m/s	2315.789474 m
79.551289 m/s	2636.836512 m
84.551289 m/s	2978.716885 m

Observation 3.2

this is the core: the crossing is the minimum.2315.789474 m at $V_1 = 74.551289$ m/s, against 2837.424143 m ten metres per second lower and 2978.716885 m ten higher——the length required is the larger of the two, so moving away from the crossing always lengthens it.

Observation 3.3

at the crossing the two distances agree.From the bisection the difference there is under 10^{-6} m——it is the one speed at which either choice needs the same length.

Observation 3.4

so V_1 is not a safety margin but the solution of a minimisation.Lowering V_1 for margin lengthens the continuing side and raises the runway length required——looking at one side only and saying “deciding early is safer” lengthens the other.

Observation 3.5

the displacement is not symmetric.Ten metres per second lower adds 521.634670 m, ten higher adds 662.927411 m——the two curves have different slopes, so the neighbourhood of the minimum is lopsided.

4. The Separator

Observation 4.1

the separator is whether the two distances move the same way with speed or opposite ways.The same way and there is no crossing, and the notion of a decision speed does not stand——the crossing exists because losing an engine costs acceleration.

Observation 4.2

beside Paper 723.Paper 723 showed that because what is carried is the fuel itself, stretches of one flight cost different amounts——there the distance was a one-directional trade. Here it is a minimum squeezed between two opposing requirements.

Observation 4.3

what is not said.Real values of V_1, the regulations, runway condition and reaction delay are not treated, and the air distance is assumed——what is shown is that V_1 is a crossing, and that the crossing is the minimum.

5. Roll Call and Closing

1. **stopping 666.666667 m at 40 m/s** (Computation 2.1): **it rises with speed.**
2. **2041.666667 m at 70 m/s** (Computation 2.1): **as the square.**
3. **continuing 3800.000000 m at 40 m/s** (Computation 2.1): **the other way.**
4. **2562.500000 m at 70 m/s** (Computation 2.1): **it has fallen.**
5. **opposite signs at the ends** (Observation 2.3): **one crossing only.**
6. **$V_1 = 74.551289$ m/s** (Computation 3.1): **found by bisection.**
7. **2315.789474 m there** (Computation 3.1): **the core.**
8. **2837.424143 m ten lower** (Computation 3.1): **it grows.**
9. **2978.716885 m ten higher** (Computation 3.1): **it grows here too.**
10. **the direction of the two distances** (Observation 4.1): **the separator; Paper 723.**

Closing: **The take-off decision speed is not a value somebody chose but the point where two distances cross, and that crossing is the minimum.The distance to stop rises with speed (666.666667 m at 40 m/s, 2041.666667 m at 70 m/s) while the distance to continue falls (3800.000000 m and 2562.500000 m at the same points)——running faster before stopping takes more room, while having reached a higher speed leaves a shorter stretch to be carried on the remaining engines.The difference is monotone, negative at 1 m/s and positive at 80 m/s, so exactly one crossing exists.Bisection gives $V_1 = 74.551289$ m/s at 2315.789474 m, the two distances differing there by under 10^{-6} m——and since the length required is the larger of the two, lowering V_1 by 10 m/s gives 2837.424143 m and raising it 2978.716885 m: either way it grows.So V_1 is not a safety margin but the solution of a minimisation, and lowering it for margin lengthens the continuing side——the two curves have different slopes, adding 521.634670 m when lowered and 662.927411 m when raised, so the neighbourhood of the minimum is lopsided.The crossing exists because losing an engine costs acceleration——were the one-out acceleration the same, the distance to continue would not depend on speed and the notion of a decision speed would not stand.The separator is whether the two distances move the same way with speed or opposite ways.Placed among the earlier papers——Paper 723 treated a one-directional trade; here it is a minimum squeezed from both sides.To be honest——real values of V_1, the regulations and reaction delay are not treated, and the air distance is assumed.**

References

1. Anderson, J. D. (1999). *Aircraft Performance and Design*. McGraw-Hill.
2. Torenbeek, E. (1982). *Synthesis of Subsonic Airplane Design*. Delft University Press.
3. Yamagishi, Y. (2026). *Because What Is Being Carried Is the Fuel Itself, the First Stretch and the Last Stretch of One Flight Cost Different Amounts* [Paper 723]. Zenodo. 10.5281/zenodo.23015015

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