

The "1/4" in Black-Hole Entropy

— Anchoring It via the 4π Solid Angle, the Euler Characteristic, and Equipartition —
and Placing the Causal Sphere on Jacobson's Horizon

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— *From thinking to syncing.* —

Positioning of this paper

This paper offers a **structural interpretation**. It does *not* give a microscopic derivation of black-hole entropy (no counting of quantum microstates, as string theory or loop quantum gravity attempt); the area law of Bekenstein and Hawking is assumed. What it does is *anchor the coefficient 1/4*: it exhibits the $1/4$ as a composite of the spatial horizon's 4π (the solid angle, $= 2\pi\chi(S^2)$ of Paper 67) and the thermal Euclidean-time 2π —which, by the discipline of Paper 58, are *distinct roots*—together with the surface-gravity factor. It also places the causal-sphere entropy of Paper 65 on Jacobson's local horizon. No claim contradicting established theory is made.

ABSTRACT

The Bekenstein–Hawking entropy $S = A/(4l_P^2)$ carries a famous coefficient, $1/4$. This paper anchors it structurally. In the first-law derivation for a Schwarzschild horizon, the Hawking temperature $T = \hbar\kappa/(2\pi ck_B)$ contributes a 2π —the period of Euclidean time, an S^1 —while the horizon area $A = 4\pi r_s^2$ contributes a 4π —the solid angle of the spatial 2 -sphere, which by Paper 67 is $2\pi\chi(S^2)$, a topological quantity. With the surface gravity $\kappa = c^4/(4GM)$, these fix the coefficient of A/l_P^2 to be exactly $1/4$. We record (Proposition 3.1) that the $1/4$ is therefore not a single mysterious number but a composite of a thermal 2π and a spatial 4π that—by the discipline of Papers 48 and 58—are *distinct roots* (imaginary-time circle vs. spatial sphere). We give the equipartition reading (Padmanabhan): $S = N/4$ with $N = A/l_P^2$ —one bit per four Planck areas. Finally, we place the causal-sphere entropy of Paper 65 ($S = \pi c^5$, exponent $5 = 2 + 3$) on Jacobson's local Rindler horizon: the causal sphere $r = ct$ of Paper 61 is such a horizon, and its $1/4$ has the same composite origin. We are explicit that this is an anchoring, not a microscopic derivation.

1. Introduction

Bekenstein argued a black hole's entropy is proportional to its horizon area; Hawking's temperature fixed the proportionality to $S = A/(4l_P^2)$, $l_P^2 = \hbar G/c^3$. The coefficient $1/4$ has since been a target: a good microscopic theory should reproduce it. String theory and loop quantum gravity count microstates to do so. This paper does something more modest and, in the spirit of the system, more structural: it *anchors* the $1/4$ —exhibits which geometric factors compose it—and places the causal-sphere entropy of Paper 65 in Jacobson's thermodynamic frame.

The two ingredients from the system are Paper 67 (the sphere's 4π is the topological $2\pi\chi(S^2)$) and Paper 65 (the causal sphere's entropy as $S = \pi c^5$ on the c -ladder).

What this paper does not do. It does not count microstates and does not derive the area law (assumed). It decomposes the $1/4$ and locates Paper 65 on a Jacobson horizon.

2. The $1/4$, Assembled

For a Schwarzschild black hole of mass M :

$$r_s = \frac{2GM}{c^2}, \quad A = 4\pi r_s^2 = \frac{16\pi G^2 M^2}{c^4}, \quad \kappa = \frac{c^4}{4GM}, \quad T_H = \frac{\hbar\kappa}{2\pi c k_B} = \frac{\hbar c^3}{8\pi G M k_B}$$

The first law $dE = T_H dS$ ($E = Mc^2$) gives

$$dS = \frac{c^2 dM}{T_H} = \frac{2\pi}{\hbar\kappa} c^2 dM = \frac{8\pi G k_B}{\hbar c^2} M dM \Rightarrow S = k_B \frac{A}{4l_P^2}$$

Proposition 3.1 (The $1/4$ is a composite of a thermal 2π and a spatial 4π)

The coefficient $1/4$ of A/l_P^2 is assembled from two geometric factors of different origin:

- a **thermal 2π** , the period of Euclidean time (the S^1 whose absence of a conical singularity fixes $T_H = \hbar\kappa/2\pi$);
- a **spatial 4π** , the solid angle of the horizon 2 -sphere, $A = 4\pi r_s^2$, which by Paper 67 is $2\pi\chi(S^2)$ —a topological quantity;

together with the surface-gravity rational factor from $\kappa = c^4/(4GM)$. The $2\pi/\kappa$ of the temperature and the 4π of the area combine so that $S = A/(4l_P^2)$.

By the discipline of Papers 48 and 58, the thermal 2π (an imaginary-time circle S^1) and the spatial 4π (a real-space sphere S^2) are **distinct roots**: different geometric objects that happen to combine in the coefficient. The " $1/4$ " is thus not one number to be explained but the residue of two.

3. The Equipartition Reading

Observation 4.1 (One bit per four Planck areas)

Writing $N = A/l_P^2$ for the number of Planck-area cells tiling the horizon, $S = \frac{1}{4} N k_B$. This is the form emphasised by Padmanabhan's equipartition of horizon degrees of freedom: the horizon carries N surface degrees of freedom, and the entropy is a quarter of their count—one bit per four Planck areas. The area law thereby sits naturally in the emergent/thermodynamic-gravity programme, the neighbourhood identified for the system.

4. The Causal Sphere on Jacobson's Horizon

Jacobson (1995) derived the Einstein field equations from the Clausius relation $\delta Q = T \delta S$ applied to *local Rindler horizons* through each spacetime point, with $S = A/(4l_P^2)$ and T the Unruh temperature. There the area law is the input and gravity is the output; the $1/4$ is exactly the coefficient that makes the thermodynamics reproduce $G_{\mu\nu} = 8\pi G T_{\mu\nu}/c^4$.

Corollary 5.1 (Paper 65 sits on a Jacobson horizon)

The causal (light) sphere $r = ct$ of Paper 61 is a local causal horizon of exactly the kind Jacobson uses. Its Bekenstein–Hawking entropy is the causal-sphere entropy of Paper 65, $S = \pi c^5$ (exponent $5 = 2 + 3$). Its coefficient $1/4$ has the composite origin of Proposition 3.1: the causal sphere is a spatial S^2 (contributing the $4\pi = 2\pi\chi$ of Paper 67), and the local Unruh temperature contributes the thermal 2π . Thus Papers 61, 65, and 67 assemble, on the causal sphere, the same $1/4$ as the black-hole horizon.

This closes the arc: the causal-sphere entropy of Paper 65 is not a stand-alone curiosity but the Bekenstein–Hawking entropy of a Jacobson horizon, with its coefficient anchored by the topological 4π of Paper 67.

5. Scope of the Claims

What is established

1. In the first-law derivation the $1/4$ of A/l_P^2 is a composite of the thermal Euclidean 2π and the spatial horizon $4\pi = 2\pi\chi(S^2)$ (Paper 67), which are distinct roots (Proposition 3.1).
2. Equivalently $S = N/4$ with $N = A/l_P^2$, one bit per four Planck areas (Observation 4.1).
3. The causal sphere of Paper 61 is a Jacobson horizon; the entropy of Paper 65 is its Bekenstein–Hawking entropy, with the same composite $1/4$ (Corollary 5.1).

What is not established / Open problem

- A microscopic derivation of black-hole entropy. No quantum microstates are counted; the area law is assumed, not derived.
- That the $1/4$ is "explained" as a microstate count would explain it. It is *decomposed* into geometric factors, which is weaker.
- Any merging of the thermal 2π and the spatial 4π . They remain distinct roots (Proposition 3.1); the composite is not a single origin.
- *Open*: is there a counting—of causal-sphere "cells," or of the Planck-area tiles of Observation 4.1—that reproduces $S = A/4$ as an actual number of states, rather than a thermodynamic coefficient? That would promote this anchoring to a microscopic derivation; it is not attempted here.

6. Conclusion

The coefficient $1/4$ of the Bekenstein–Hawking entropy is not a single mysterious number. In the first-law derivation it is assembled from a thermal 2π (the Euclidean-time circle) and a spatial 4π (the horizon sphere's solid angle, $= 2\pi\chi(S^2)$ by Paper 67), together with the surface-gravity factor; the two geometric inputs are distinct roots. Read through equipartition, $S = N/4$ is one bit per four Planck areas. The causal sphere of Paper 61 is a Jacobson horizon, so the causal-sphere entropy of Paper 65 is its Bekenstein–Hawking entropy, its $1/4$ anchored by the topological 4π of Paper 67. **This is a structural anchoring in the emergent-gravity neighbourhood, not a microscopic derivation**—the honest thing the system can say about the most famous number in horizon thermodynamics.

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