Regeneration's Value Rides on the Square of Speed, So Leaving the Last Low-Speed Stretch to Friction Costs Under One Percent

— what remains below 10 km/h is only 0.010000 of the energy at 100 km/h, so with a round-trip efficiency of 0.70 giving the low-speed stretch to friction loses only 0.007000 of the whole — the separator is whether the brake turns the energy into heat or into charge — [Paper 735]

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— From thinking to syncing. —

Abstract

A regenerative brake recovers energy. Where it can be recovered is settled by the square of speed.The energy in each band of speed is counted, and with it what may be given up.No new theorem or law is claimed. First, the energy goes as the square of speed——100 km/h holds exactly 4.000000 times what 50 km/h holds (Section 2). **Second, and this is the core. Almost nothing remains at low speed——below 10 km/h there is only 0.010000 of the energy at 100 km/h** (Section 2). **Third, so giving the low-speed stretch to friction costs little——at a round-trip efficiency of 0.70, the loss is 0.007000 of the whole** (Section 3). **Fourth, one band at the fast end carries the weight——from 70 to 100 km/h alone accounts for 0.510000** (Section 3). **Fifth, everything up to 50 km/h is a quarter——accumulated, it comes to only 0.250000** (Section 3). **Sixth, the separator is whether the brake turns the energy into heat or into charge——the distance to stop is the same; the destination differs** (Section 4). **Keywords**: regenerative braking, kinetic energy, square of speed, round-trip efficiency, share by band of speed, heat or charge.

Scope of this paper (scope note): **No new theorem or law is claimed——that kinetic energy goes as the square of speed, that regenerative braking loses to round-trip efficiency, and that friction braking takes over at low speed are all standard (Genta 1997, Ehsani 2018).No precision is claimed for representative values——a round-trip efficiency of 0.70 and a mass of 1500 kg are order-of-magnitude values.The battery side is not counted——that cold or a full charge prevents regeneration is mentioned only.Road load is not included——deceleration from rolling and aerodynamic resistance is not included; only kinetic energy is considered.No real vehicle is claimed——particular vehicles and control designs are not treated.Gradients are not treated——recovery of potential energy on a descent is not treated.Relation to earlier papers**: **Paper 725 showed that the take-off decision speed is the crossing of two distances and that the crossing is the minimum——there braking was a quantity that makes distance, and the length needed to stop was the subject. Here the same braking is counted for what it discards. The distance needed, and the energy lost.**What is added is **tabulating the energy against speed**, **putting the low-speed share at 0.010000**, **putting the loss from friction at 0.007000**, **showing one fast band to hold 0.510000**, and **placing the separator on heat against charge**.

1. Notation and the Question

Setting: **kinetic energy is half the mass times the square of speed. Regeneration recovers a part of it.**

$$E = \tfrac{1}{2} m v^2, \qquad E_{\text{recovered}} = \eta\, E, \qquad \frac{E(v_1)}{E(v_2)} = \left(\frac{v_1}{v_2}\right)^2$$

A mass of 1500 kg and a round-trip efficiency $\eta = 0.70$ are taken as representative.

Question: **where does the value of chasing regeneration lie?**

2. Almost Nothing Remains at Low Speed

Computation 2.1: **the kinetic energy was derived for each speed.**

speed	energy	in kWh
100 km/h	578703.703704 J	0.160751
50 km/h	144675.925926 J	0.040188
30 km/h	52083.333333 J	0.014468
10 km/h	5787.037037 J	0.001608
5 km/h	1446.759259 J	0.000402

> **Observation 2.2**
>
> 100 km/h holds exactly 4.000000 times what 50 km/h holds.Tripling the speed gives nine times——being a square, it tilts sharply toward the fast end.

> **Observation 2.3**
>
> this is the core: below 10 km/h there is only 0.010000.A tenth of the speed of 100 km/h leaves the square of a tenth——the stretch just before stopping is, on the energy ledger, nearly empty.

3. The Value Is Concentrated at the Fast End

Computation 3.1: taking 100 km/h as one, the share in each band of speed was derived.

band	share of that band	accumulated
0 to 10 km/h	0.010000	0.010000
10 to 30 km/h	0.080000	0.090000
30 to 50 km/h	0.160000	0.250000
50 to 70 km/h	0.240000	0.490000
70 to 100 km/h	0.510000	1.000000

Observation 3.2

the band from 70 to 100 km/h alone holds 0.510000.Everything up to 50 km/h together holds only 0.250000——the value of regeneration is concentrated at the fast end.

Observation 3.3

so a design that gives low speed to friction stands.At a round-trip efficiency of 0.70, the loss is 0.007000 of the whole——giving it up to simplify the control costs under one percent.

Computation 3.4: **the energy recoverable in stopping from 100 and from 50 km/h was derived.**

initial speed	recovered	in kWh
100 km/h	405092.592593 J	0.112526
50 km/h	101273.148148 J	0.028131

Observation 3.5

one deceleration recovers little in town.Stopping from 50 km/h yields 0.028131 kWh——but there are many of them, so in town driving it accumulates.

Observation 3.6

the battery side carries a different limit.Cold or a full charge cannot accept it——this paper does not count that, but it is worth writing that regeneration stops for reasons unrelated to the square.

4. The Separator

> **Observation 4.1**
>
> **the separator is whether the brake turns the energy into heat or into charge.The distance to stop is the same; the destination differs——under the one word “braking”, losing and storing stand apart.**

> **Observation 4.2**
>
> **beside Paper 725.Paper 725 showed the take-off decision speed to be the crossing of two distances——there braking was a quantity that makes distance. Here it is a quantity that discards energy. The distance needed, and the energy lost.**

> **Observation 4.3**
>
> **what is not said.Battery acceptance, road load, real vehicles and gradients are not treated, and two representative values are held fixed——what is shown is the distribution of value that follows from one square.**

5. Roll Call and Closing

1. **578703.703704 J at 100 km/h** (Computation 2.1): **0.160751 kWh.**
2. **144675.925926 J at 50 km/h** (Computation 2.1): **a quarter.**
3. **a ratio of exactly 4.000000** (Observation 2.2): **it is a square.**
4. **5787.037037 J at 10 km/h** (Computation 2.1): **nearly nothing.**
5. **a share of 0.010000** (Observation 2.3): **the core.**
6. **loss to friction 0.007000** (Observation 3.3): **under one percent.**
7. **0.510000 in one band** (Computation 3.1): **over half.**
8. **0.250000 up to 50** (Computation 3.1): **a quarter.**
9. **405092.592593 J recovered** (Computation 3.4): **from 100 km/h.**
10. **heat or charge** (Observation 4.1): **the separator; Paper 725.**

Closing: **The value of chasing regeneration rides on the square of speed, so giving the last low-speed stretch to friction costs under one percent.A car of 1500 kg holds 578703.703704 J (0.160751 kWh) at 100 km/h, 144675.925926 J at 50 km/h and only 5787.037037 J at 10 km/h, with 100 km/h holding exactly 4.000000 times 50 km/h——being a square it tilts sharply toward the fast end, and below 10 km/h only 0.010000 remains.So at a round-trip efficiency of 0.70, giving the low-speed stretch to friction loses only 0.007000 of the whole——giving it up to simplify the control costs under one percent.Conversely the band from 70 to 100 km/h alone holds 0.510000, while everything up to 50 km/h together holds only 0.250000——the value of regeneration is concentrated at the fast end.One deceleration recovers little in itself: 405092.592593 J (0.112526 kWh) from 100 km/h and 0.028131 kWh from 50——but there are many of them, so in town driving it accumulates.The battery side carries a different limit, being unable to accept when cold or fully charged——this paper does not count that, but regeneration stops for reasons unrelated to the square.The separator is whether the brake turns the energy into heat or into charge.Placed among the earlier papers——in Paper 725 braking was a quantity that makes distance; here it is one that discards energy.To be honest——battery acceptance, road load and gradients are not treated, and two representative values are held fixed.**

References

1. Ehsani, M., Gao, Y., Longo, S., & Ebrahimi, K. (2018). *Modern Electric, Hybrid Electric, and Fuel Cell Vehicles*. CRC Press.
2. Genta, G. (1997). *Motor Vehicle Dynamics*. World Scientific.
3. Yamagishi, Y. (2026). *The Take-off Decision Speed Is Not Chosen but Where Two Distances Cross, and That Crossing Is the Minimum* [Paper 725]. Zenodo. 10.5281/zenodo.23015025

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