Order Variability Comes from How Demand Is Read, and a Bookkeeping Convention Alone Shifts the Amplification by Forty Percent

— with the same demand, lead time and window, the convention for how many periods the target covers alone splits the variance amplification into **2.920000** and **2.120000**, a ratio of **1.377358** — the separator is whether the amplification comes from the demand or from the reading of it — [Paper 736]

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— From thinking to syncing. —

Abstract

Orders swing more violently the further upstream one looks. What swings is not the demand.The amplification is counted from the policy, and the bookkeeping convention inside that policy is shown to change it by forty percent.No new theorem or law is claimed. First, the demand does not swing——independent random numbers of mean 100 and standard deviation 10, with no autocorrelation and no seasonality (Section 2). **Second, orders amplify all the same——the measured amplification of variance per stage is 2.918477** (Section 2). **Third, stages compound——through four, the variance grows 172.215749 times, or 13.123100 in standard deviation** (Section 2). **Fourth, and this is the core. There are two closed forms——counting the target over $L+1$ periods gives 2.920000; over L periods it gives 2.120000** (Section 3). **Fifth, the measurement picks one of them——it agrees with the convention used to 1.523×10^{-3} and differs from the other by 1.377358** (Section 3). **Sixth, the separator is whether the amplification comes from the demand or from the reading of it——it comes from the reading, so figures cannot be compared without matching the reading down to its details** (Section 4). **Keywords**: bullwhip effect, variance of orders, moving average, lead time, bookkeeping convention, demand or reading.

Scope of this paper (scope note): **No new theorem or law is claimed——that order variance grows upstream, that moving averages and replenishment policies are part of the cause, and that the amplification follows from lead time and window length are all standard (Forrester 1961, Chen 2000).Demand is kept simple——independent normal draws, with seasonality, autocorrelation, stockouts, quantity discounts and price changes not treated.Countermeasures are not designed——information sharing and demand visibility are not treated.One reading only——only the moving average is treated; exponential smoothing is not.No real supply chain is claimed——figures for particular firms or sectors are not treated.** ⚠️ **Which convention is correct is not claimed——whether the target covers L or $L+1$ periods is a definition either way. What is claimed is only that they must be matched before figures are compared.Relation to earlier papers**: **Paper 549 showed that the time inside changes almost sixfold and the product does not move——there the subject was a relation between quantities that does not depend on policy. Here, in the same inventory system, the variance of orders depends strongly on policy. A relation independent of policy, and a quantity fixed by it.**What is added is **measuring the amplification by simulation**, **putting the four-stage compounding at 172.215749**, **showing that two closed forms exist**, **confirming to 10^{-3} which one the measurement matches**, and **placing the separator on demand against reading**.

1. Notation and the Question

Setting: **each stage forms a target stock from a moving average of the last N periods and passes the change upward on top of the demand it saw. How many periods the target covers is the bookkeeping convention.**

$$\bar{d}_t = \frac{1}{N}\sum_{i=0}^{N-1} d_{t-i}, \qquad S_t = c\,\bar{d}_t, \qquad o_t = d_t + (S_t - S_{t-1})$$

c is the number of periods the target covers, taken as L or $L+1$. Here $N=5$, $L=2$, and demand is independent with mean 100 and standard deviation 10 over 400000 periods.

Question: **do upstream orders swing because the demand swings?**

2. The Demand Does Not Swing, and Orders Amplify

Computation 2.1: **the same policy was stacked four deep and the variance of orders measured at each stage.**

stage	variance of orders	against the previous
demand	100.135772	—
stage 1	292.243905	2.918477
stage 2	1037.933142	3.551599
stage 3	4110.264104	3.960047
stage 4	17244.956942	4.195584

Observation 2.2

the demand has no autocorrelation and no seasonality.Independent draws go in, and the variance of orders rises 2.918477 times in one stage——the amplification is not a property of the demand but of how the demand is read.

Observation 2.3

stages compound.Through four, the variance grows 172.215749 times, or 13.123100 in standard deviation ——the per-stage ratio grows upward because the incoming series is no longer independent.

3. There Are Two Closed Forms

Computation 3.1: **the order is $o_t = d_t + c(d_t - d_{t-N})/N$, so the amplification is $(1 + c/N)^2 + (c/N)^2$. It was derived under both conventions.**

target covers	c/N	amplification per stage
$L + 1 = 3$ periods	0.6	2.920000
$L = 2$ periods	0.4	2.120000

Observation 3.2

this is the core: the same demand, lead time and window split into two values.The ratio is 1.377358——what differs is the bookkeeping convention alone, and nothing in the world has changed.

Observation 3.3

the measurement agrees with the convention used.The measured 2.918477 agrees with the $L+1$ form's 2.920000 to 1.523×10^{-3} and does not agree with the L form's 2.120000——the simulation names which convention was used.

Observation 3.4

the L form is the one usually quoted.Expanding $1+2L/N+2L^2/N^2$ gives $(1+L/N)^2+(L/N)^2$, agreeing to within 10^{-15}——so quoting "the amplification is 2.120000" is a value under the convention that the target covers L periods.

Computation 3.5: **the amplification was derived for other windows of the moving average.**

window	per stage	through four
2 periods	**5.000000**	**625.000000**
5 periods	**2.120000**	**20.199631**
10 periods	**1.480000**	**4.797852**
20 periods	**1.220000**	**2.215335**

Observation 3.6

a wider window lowers the amplification.5.000000 per stage at two periods and 1.220000 at twenty——but a wider window is slower to follow a change in demand, so this is a trade and not a solution.

4. The Separator

Observation 4.1

the separator is whether the amplification comes from the demand or from the reading of it.It comes from the reading, so figures cannot be compared without matching the reading down to its details——“our amplification is such-and-such” means nothing without the policy and the convention attached.

Observation 4.2

beside Paper 549.Paper 549 showed that the time inside changes almost sixfold and the product does not move——there a relation between quantities did not depend on policy. Here the variance of orders depends on policy, and on the details of the convention. A relation independent of policy, and a quantity fixed by it.

Observation 4.3

what is not said.Seasonality, stockouts, countermeasures and real supply chains are not treated, and which convention is correct is not claimed——what is shown is where the amplification comes from and that its value depends on a convention.

5. Roll Call and Closing

1. **demand variance $\mathbf{100.135772}$** (Computation 2.1): **independent draws.**
2. **$\mathbf{292.243905}$ at stage $\mathbf{1}$** (Computation 2.1): **$\mathbf{2.918477}$ times.**
3. **$\mathbf{17244.956942}$ at stage $\mathbf{4}$** (Computation 2.1): **it compounds.**
4. **$\mathbf{172.215749}$ through four** (Observation 2.3): **in variance.**
5. **$\mathbf{13.123100}$ in deviation** (Observation 2.3): **the same fact.**
6. **the $L+1$ form gives $\mathbf{2.920000}$** (Computation 3.1): **the core.**
7. **the L form gives $\mathbf{2.120000}$** (Computation 3.1): **the quoted one.**
8. **a ratio of $\mathbf{1.377358}$** (Observation 3.2): **convention alone.**
9. **agreement to $\mathbf{1.523} \times 10^{-3}$** (Observation 3.3): **it names the convention.**
10. **demand or reading** (Observation 4.1): **the separator; Paper 549.**

Closing: **The variability of orders comes from the reading, not the demand; and the bookkeeping convention alone changes the amplification by forty percent.Putting independent demand of mean 100 and standard deviation 10 through a policy with a five-period moving average and a lead time of two, the measured amplification of variance per stage is 2.918477, and through four stages the variance grows 172.215749 times, or 13.123100 in standard deviation——the demand has no autocorrelation and no seasonality, so the amplification comes from how the demand is read.And there are two closed forms: counting the target over $L+1=3$ periods gives 2.920000, over $L=2$ periods gives 2.120000, a ratio of 1.377358——what differs is the bookkeeping convention alone, and nothing in the world has changed.The measurement agrees with the convention used to 1.523×10^{-3} and not with the other——the simulation names which convention was used.The L form is the one usually quoted, expanding to $(1+L/N)^2+(L/N)^2$ within 10^{-15}, so "the amplification is 2.120000" is a value under that convention.A wider window lowers the amplification, 5.000000 per stage at two periods against 1.220000 at twenty——but a wider window follows a change in demand more slowly, so this is a trade and not a solution.The separator is whether the amplification comes from the demand or from the reading of it.Placed among the earlier papers——in Paper 549 a relation did not depend on policy; here a quantity depends on policy and on the details of its convention.To be honest——seasonality, stockouts and countermeasures are not treated, and which convention is correct is not claimed.**

References

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