A Sharpe Ratio May Be Annualised by the Square Root of 252 Only When Daily Returns Are Unrelated to One Another

— with a daily autocorrelation ρ, the ratio of the correct annual value to the $\sqrt{252}$ figure is 0.734761 at $\rho = +0.3$ and 1.360991 at $\rho = -0.3$ — the separator is the sign of the autocorrelation — [Paper 738]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

Abstract

Annualising a daily figure by a square root is used as a matter of course. It rests on the convention that daily returns are unrelated to one another.Which way, and how far, it errs when that convention fails is counted.No new theorem or law is claimed. First, under the convention the annual ratio is 0.634980——the daily 0.04 times $\sqrt{252} = 15.874508$ (Section 2). **Second, and this is the core. Positive autocorrelation overstates and negative understates——a ratio of 0.734761 at $\rho = +0.3$ and 1.360991 at $\rho = -0.3$** (Section 2). **Third, the annual figures spread apart——the same daily 0.04 becomes 0.466559 or 0.864202** (Section 2). **Fourth, annualising from finer periods errs more——at $\rho = +0.3$, 0.802314 from 4 periods a year and 0.734761 from 252** (Section 3). **Fifth, simulation confirms it——over 8000 years the ratios are 0.740289 and 1.341098, within 3 % of the exact expression** (Section 3). **Sixth, the separator is the sign of the autocorrelation——the sign sets the direction of the error; its size and the period set the amount** (Section 4). **Keywords**: Sharpe ratio, annualisation, autocorrelation, first-order autoregression, direction of the error, fineness of the period.

Scope of this paper (scope note): **No new theorem or law is claimed——the Sharpe ratio, its annualisation and the correction for serial correlation are all known (Sharpe 1966, Lo 2002).One model only——daily returns are taken as first-order autoregressive; fat tails, changing variance and non-stationarity are not treated.Error in the mean is not treated——the ratio is taken from variances, and sampling variation of the mean is left out.No claim about autocorrelation at each period——placing the same ρ however the year is divided is a choice made for comparison, not a claim that daily and quarterly autocorrelations are equal.No real strategy is claimed——particular strategies and markets are not treated.Relation to earlier papers**: **Paper 706 showed that a backtest measures not the quality of a strategy but the number of attempts——its scope note stated that the standard error of the annual Sharpe ratio was fixed at a rule of thumb, with autocorrelation and fat tails not included. Here that autocorrelation is included. The appearance produced by the number of attempts, and the appearance produced by the way of measuring.**What is added is **the exact ratio for each autocorrelation**, **showing its direction to flip with the sign**, **showing that the period one annualises from changes the error**, **confirming it within 3 % by simulation**, and **placing the separator on the sign of the autocorrelation**.

1. Notation and the Question

Setting: **let daily returns have mean μ, standard deviation σ and first-order autocorrelation ρ. With q days to a year, the variance of the annual sum is not $q\sigma^2$ but the following.**

$$\operatorname{Var}\Big(\sum_{t=1}^{q} r_t\Big) = \sigma^2\Big(q + 2\sum_{k=1}^{q-1}(q-k)\rho^k\Big), \qquad \frac{\text{correct annual}}{\sqrt{q}\ \text{times}} = \sqrt{\frac{q}{q + 2\sum_{k=1}^{q-1}(q-k)\rho^k}}$$

For large q this ratio approaches $\sqrt{(1-\rho)/(1+\rho)}$.

Question: **what convention does square-root annualisation rest on?**

2. The Sign of the Autocorrelation Sets the Direction

Computation 2.1: **with a daily ratio of 0.04, the ratio and the correct annual value were derived for each autocorrelation (exact at $q = 252$, and the limit).**

ρ	ratio (exact)	ratio (limit)	correct annual ratio
-0.3	1.360991	1.362770	0.864202
-0.1	1.105099	1.105542	0.701716
0.0	1.000000	1.000000	0.634980
$+0.1$	0.904897	0.904534	0.574592
$+0.3$	0.734761	0.733799	0.466559
$+0.5$	0.578884	0.577350	0.367580

Observation 2.2

this is the core: one operation overstates under positive autocorrelation and understates under negative.At $\rho = +0.3$ the square-root figure must be shrunk to 0.734761; at $\rho = -0.3$ stretched to 1.360991——the same daily 0.04 becomes 0.466559 or 0.864202 a year.

Observation 2.3

positive autocorrelation piles small daily swings into large annual ones.The annual variance exceeds $q\sigma^2$, so the annual ratio falls below the square-root figure——negative autocorrelation instead lets the swings cancel.

Observation 2.4

the exact value is close to the limit.At $\rho = +0.3$ they differ by 9.619×10^{-4}——252 days is long enough for this ratio.

3. Annualising from Finer Periods Errs More

Computation 3.1: holding $\rho = +0.3$, the number of periods in a year was varied.

periods per year	ratio
4	0.802314
12	0.754829
52	0.738496
252	0.734761

Observation 3.2

at the same ρ, annualising from finer periods errs more.0.802314 from 4 periods a year, 0.734761 from 252 ——the more terms pile up, the more the autocorrelation accumulates.

Computation 3.3: daily returns of mean 0.0004 and standard deviation 0.01 were generated for 8000 years and the ratio taken from the variance of annual sums.

ρ	simulation	exact	relative difference
0.0	1.011820	1.000000	0.011820
$+0.3$	0.740289	0.734761	0.007523
-0.3	1.341098	1.360991	0.014616

Observation 3.4

all three agree with the exact expression within 3 %.Taking the ratio from variances keeps the sampling variation of the mean out——direction and size are both confirmed by a separate route.

4. The Separator

Observation 4.1

the separator is the sign of the autocorrelation.The sign sets the direction of the error; its size and the period set the amount——annual ratios must be compared from the same period, with the autocorrelation attached.

Observation 4.2

beside Paper 706.Paper 706 showed that a backtest measures the number of attempts, and stated that autocorrelation was not included——there the number of attempts produced an appearance. Here the way of annualising does.

Observation 4.3

what is not said.Fat tails, changing variance, error in the mean and real strategies are not treated, and the same ρ is placed at every period——what is shown is the convention square-root annualisation rests on, and the direction in which it fails.

5. Roll Call and Closing

1. $\sqrt{252} = \mathbf{15.874508}$ (Section 1): **the annualising factor.**
2. **annual $\mathbf{0.634980}$** (Computation 2.1): **under the convention.**
3. **ratio $\mathbf{0.734761}$ at $\rho = +0.3$** (Computation 2.1): **the core.**
4. **$\mathbf{1.360991}$ at $\rho = -0.3$** (Computation 2.1): **the other way.**
5. **annual $\mathbf{0.466559}$ and $\mathbf{0.864202}$** (Observation 2.2): **from one daily figure.**
6. **limit $\mathbf{0.733799}$** (Computation 2.1): **close to exact.**
7. **$\mathbf{0.802314}$ from 4 periods** (Computation 3.1): **a smaller error.**
8. **simulation $\mathbf{0.740289}$** (Computation 3.3): **at $\rho = +0.3$.**
9. **simulation $\mathbf{1.341098}$** (Computation 3.3): **at $\rho = -0.3$.**
10. **the sign of the autocorrelation** (Observation 4.1): **the separator; Paper 706.**

Closing: **A Sharpe ratio may be annualised by the square root of 252 only when daily returns are unrelated to one another.A daily ratio of 0.04 times $\sqrt{252} = 15.874508$ gives 0.634980, resting on the convention that annual variance is 252 times daily variance.With daily autocorrelation the correct ratio is 0.734761 of that at $\rho = +0.3$ and 1.360991 at $\rho = -0.3$, so one daily 0.04 becomes 0.466559 or 0.864202 a year——positive autocorrelation piles daily swings into annual ones and negative lets them cancel, so one operation overstates or understates by sign.And at the same $\rho = +0.3$, annualising from 4 periods gives 0.802314 while 252 gives 0.734761: finer periods err more.Simulating 8000 years gave 0.740289 and 1.341098, within 3 % of the exact expression.The separator is the sign of the autocorrelation——annual ratios must be compared from the same period, with the autocorrelation attached.Placed among the earlier papers——Paper 706 treated the appearance made by the number of attempts and left autocorrelation out; this is the appearance made by the way of measuring.To be honest——the model is first-order autoregressive, and fat tails, changing variance and error in the mean are not treated.**

References

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