What Sets the Length of a Transition Curve Swaps When Cant Is Added

— with cant the jerk needs a transition of only **47.094262** m, and the **100.000000** m required by the rate of raising the cant decides — the separator is whether the length is set by the rate of change of uncompensated lateral acceleration or by the rate of raising the cant itself — [Paper 741]

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— *From thinking to syncing.* —

Abstract

A railway curve does not pass straight from tangent to circle. A transition curve, whose curvature grows gradually, is placed between them.What sets its length is counted with cant and without.No new theorem or law is claimed. First, without cant, jerk sets the length——154.320988 m at 120 km/h, and the ratio to 130 km/h is the cube of speed, 1.271412 (Section 3). **Second, cant stops at its limit——the balancing cant at 120 km/h, 0.151116 m, exceeds the limit 0.105 m, leaving 0.423848 m/s² uncompensated** (Section 2). **Third, and this is the core. With cant, the constraint that sets the length swaps——jerk needs only 47.094262 m, but raising the cant needs 100.000000 m** (Section 3). **Fourth, close lengths can have different reasons——at 80 km/h, 45.724737 m without cant and 42.643041 m with** (Section 3). **Fifth, a transition shifts the circle——over 100 m the end heading is 0.062500 rad and the circle moves 0.520761 m inward** (Section 4). **Sixth, the separator is whether the length is set by the rate of change of uncompensated lateral acceleration or by the rate of raising the cant itself——"how much transition is needed?" does not say which constraint it is about until one says whether cant is used** (Section 5). **Keywords**: transition curve, cant, jerk, lateral acceleration, clothoid, swap of the binding constraint.

Scope of this paper (scope note): **No new theorem or law is claimed——balancing cant, the lateral acceleration cant cannot cancel, transition lengths from jerk and from cant-raising rate, the curve whose curvature is proportional to arc length (the clothoid) and the approximation for the circle's shift are all known (Esveld 2001, Levien 2008).No design standard is made——gauge, cant limit, allowed jerk and cant-raising limit are all representative values; actual standards differ by country and line.Vehicles are not treated——body roll, bogies, springs and track irregularities are not treated; only the lateral acceleration of a point mass is considered.One way of raising the cant——cant is taken to rise at a uniform rate along the transition.No real line is claimed——particular lines and trains are not treated.Roads are not treated——road superelevation and friction belong to Paper 453.Relation to earlier papers**: **Paper 453 showed that on an icy banked curve, driving slowly can be more dangerous than driving fast——its scope note stated that design standards, transition curves and sight distance were not the subject. There the balance of forces standing inside the curve was treated. Here the way into the curve is treated: over how many metres that balance is built up. The balance inside the curve, and the way into it.**What is added is **deriving lengths from two constraints at four speeds**, **showing that the binding constraint swaps with cant**, **showing that at 80 km/h the lengths are close for different reasons**, **deriving the circle's shift by numerical integration and by the approximation**, and **placing the separator on the rate of change of lateral acceleration against the rate of raising cant**.

1. Notation and the Question

Setting: **let gauge be G, radius R and speed v. Cant C is the height by which the outer rail is raised, cancelling part of the lateral acceleration. Along the transition, the uncompensated lateral acceleration a must grow no faster than an allowed jerk J, or the cant must rise no faster than a limit $\dot{C}$; each gives a required length.**

$$C_{\text{eq}} = \frac{Gv^2}{gR}, \qquad a = \frac{v^2}{R} - \frac{gC}{G}, \qquad L_J = \frac{a\,v}{J}, \qquad L_C = \frac{C\,v}{\dot{C}}$$

Take $G = 1.067$ m, $R = 800$ m, a cant limit of 0.105 m, $J = 0.3$ m/s^3 and $\dot{C} = 0.035$ m/s. Without cant, $a = v^2/R$.

Question: **what sets the length of a transition curve?**

2. Cant Stops at Its Limit

Computation 2.1: **at four speeds, the balancing cant, the cant actually applied (capped at the limit) and the uncompensated lateral acceleration were derived.**

speed	balancing cant	applied cant	uncompensated acceleration
80 km/h	0.067163 m	0.067163 m	0.000000 m/s^2
100 km/h	0.104942 m	0.104942 m	0.000000 m/s^2
120 km/h	0.151116 m	0.105000 m	0.423848 m/s^2
130 km/h	0.177352 m	0.105000 m	0.664975 m/s^2

Observation 2.2

up to 100 km/h cant cancels the lateral acceleration fully; from 120 km/h it stops at the limit and some remains.At 120 km/h, 1.388889 m/s^2 without cant, 0.423848 m/s^2 with it——cant removes lateral acceleration, but only up to a limit.

3. The Binding Constraint Swaps

Computation 3.1: **at the same four speeds, transition lengths were derived without cant (from jerk) and with cant (from both constraints).**

speed	no cant (jerk)	with cant (jerk)	with cant (cant-raising)
80 km/h	45.724737 m	0.000000 m	42.643041 m
100 km/h	89.306127 m	0.000000 m	83.287190 m
120 km/h	154.320988 m	47.094262 m	100.000000 m
130 km/h	196.205561 m	80.043275 m	108.333333 m

Observation 3.2

without cant, jerk sets the length, which grows as the cube of speed.The ratio of 130 to 120 km/h is 1.271412 , equal to $(130/120)^3$——lateral acceleration goes as speed squared, and the time to build it up goes inversely with speed.

Observation 3.3

this is the core: with cant, the constraint that sets the length swaps.At 120 km/h jerk needs only 47.094262 m, but raising the cant to 0.105000 m needs 100.000000 m, and that decides——cant relaxes the jerk constraint, but brings in a new one: raising the cant itself.

Observation 3.4

close lengths, different reasons.At 80 km/h the length is 45.724737 m without cant and 42.643041 m with it; the first is set by jerk, the second by the cant-raising rate, and the jerk constraint with cant is 0.000000 m and idle——the length alone does not tell which constraint is acting.

4. A Transition Shifts the Circle

Computation 4.1: **a curve 100 m long, whose curvature rises in proportion to arc length from 0 to $1/R$, was integrated numerically by Simpson's rule.**

quantity	numerical integration	closed form or approximation
end heading	**0.062500** rad	$L/(2R) = 0.062500$
shift of the circle	**0.520761** m	$L^2/(24R) = 0.520833$

Observation 4.2

with a transition inserted, the circle must move 0.520761 m inward to connect with the tangent.The relative difference from $L^2/(24R)$ is 1.395×10^{-4}——a curve is not fixed by its radius alone; the length of the way in moves the circle itself.

5. The Separator

Observation 5.1

the separator is whether the length is set by the rate of change of uncompensated lateral acceleration (jerk) or by the rate of raising the cant itself.Without cant the former decides; with cant, the latter——"how much transition is needed?" does not say which constraint it is about until one says whether cant is used.

Observation 5.2

beside Paper 453.Paper 453 treated the balance of forces standing inside a banked curve and stated that transition curves were not the subject——this is its entrance, counting over how many metres the balance is built. The balance inside the curve, and the way into it.

Observation 5.3

what is not said.All values are representative; design standards, vehicle motion and real lines are not treated ——what is shown is that the constraint setting the length swaps with cant, and that the circle shifts.

6. Roll Call and Closing

1. **balancing cant $\mathbf{0.151116}$ m** (Computation 2.1): **over the limit at 120 km/h.**
2. **uncompensated $\mathbf{0.423848}$ m/s^2** (Computation 2.1): **what the capped cant leaves.**
3. **no cant $\mathbf{154.320988}$ m** (Computation 3.1): **jerk decides.**
4. **ratio $\mathbf{1.271412}$** (Observation 3.2): **the cube of speed.**
5. **from jerk $\mathbf{47.094262}$ m** (Computation 3.1): **relaxed by cant.**
6. **from cant-raising $\mathbf{100.000000}$ m** (Computation 3.1): **the core.**
7. **$\mathbf{45.724737}$ m and $\mathbf{42.643041}$ m** (Observation 3.4): **close, for different reasons.**
8. **end heading 0.062500 rad** (Computation 4.1): **$L/(2R)$.**
9. **shift $\mathbf{0.520761}$ m** (Computation 4.1): **the way in moves the circle.**
10. **lateral-acceleration rate or cant-raising rate** (Observation 5.1): **the separator; Paper 453.**

Closing: **What sets the length of a transition curve swaps when cant is added.With gauge 1.067 m and radius 800 m, without cant jerk sets the length, 154.320988 m at 120 km/h, and the ratio to 130 km/h is the cube of speed, 1.271412——because lateral acceleration goes as speed squared.With cant, the balancing cant at 120 km/h, 0.151116 m, stops at the limit 0.105 m, and the uncompensated lateral acceleration falls to 0.423848 m/s^2.Jerk then needs only 47.094262 m, but raising the cant needs 100.000000 m, and that decides——cant relaxes the jerk constraint but brings in the constraint of raising the cant.At 80 km/h the lengths are close, 45.724737 m and 42.643041 m, yet set by different constraints.Inserting 100 m of transition moves the circle 0.520761 m inward, so a curve is not fixed by its radius alone.The separator is whether the length is set by the rate of change of uncompensated lateral acceleration or by the rate of raising the cant itself.Placed among the earlier papers——Paper 453 treated the balance inside the curve and set transition curves aside; this counts the way into the curve.To be honest——all values are representative, design standards, vehicles and real lines are not treated, and cant is taken to rise uniformly.**

References

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