A Railway's Hauling Margin Is Large Only on the Flat; Once Grade Enters the Denominator, Rubber Tyres Win

— Taking the weight on driving wheels as **1**, the maximum trailing load on the flat is **165.666667** on dry rail and **69.000000** on rubber tyres, rail ahead by **2.400966** times — the separator is whether the margin is set by the size of adhesion or the smallness of rolling resistance — [Paper 747]

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— From thinking to syncing. —

Abstract

Steel wheels on rail have small rolling resistance, so heavy trains can be hauled with little power. But the maximum load is also set by the force the wheels can push without slipping.That limit is counted for dry rail, wet rail and rubber tyres as the grade varies.No new theorem or law is claimed. First, on the flat, rail wins by far——dry rail 165.666667, rubber tyres 69.000000, 2.400966 times (Section 2). **Second, that win comes not from adhesion but from a small denominator——rubber has more adhesion, but rail's rolling resistance of 0.0015 is far below rubber's 0.010** (Section 2). **Third, and this is the core. When grade is added to the same denominator, rail's advantage vanishes——at 1 % the denominator grows 7.666667 times for rail and 2.000000 for rubber, limits 20.739130 against 34.000000** (Section 3). **Fourth, the grade at which they are equal is small——0.322222 %, where the limit is 51.941176** (Section 3). **Fifth, wet rail loses to rubber even on the flat——65.666667 against 69.000000, and 0.941748 at a 5 % grade** (Section 3). **Sixth, the separator is whether the margin is set by the size of adhesion or the smallness of rolling resistance——"railways haul heavy loads" does not say which hauls more until the grade is stated** (Section 4). **Keywords**: adhesion coefficient, rolling resistance, grade resistance, hauling limit, rail and rubber tyres, smallness of the denominator.

Scope of this paper (scope note): **No new theorem or law is claimed——the tractive limit set by adhesion, adding rolling and grade resistance, and that railways are weak on grades while rubber-tyred vehicles climb well are all known (Hay 1982, Wong 2008).Only the limit——power, braking, multiple locomotives and distributed traction are not treated; only the limit set by pushing without slipping is considered.Coefficients are representative——adhesion coefficients of 0.25, 0.10 and 0.70 and rolling resistance coefficients of 0.0015 and 0.010 are for comparison and move greatly with surface and weather.Other resistances are not included——air resistance, curve resistance and acceleration are not included.No single cause——why railways avoid grades is not reduced to this; it is shown as one factor seen from the adhesion limit.No real vehicle is claimed——particular trains and trucks are not treated.Relation to earlier papers**: **Paper 667 showed that rolling resistance is not friction, vanishing for a perfectly elastic wheel and independent of speed——its scope note stated that climbing, acceleration and drivetrain losses would not be included, only steady running on the flat. There the nature of rolling resistance was treated. Here its smallness sets the load that can be hauled, and adding grade to the same denominator erases the advantage of that smallness. The nature of rolling resistance, and the value of its smallness.**What is added is **tabulating limits by grade for three surfaces**, **showing that at 1 % the denominator grows 7.666667 times for rail and 2.000000 for rubber**, **deriving the equal grade 0.322222 %**, and **placing the separator on adhesion against rolling resistance**.

1. Notation and the Question

Setting: **let the weight on driving wheels be W_d, the trailing weight W_t, the adhesion coefficient μ, the rolling resistance coefficient C_{rr}, and the grade the tangent g. The limit is reached when the force the driving wheels can push without slipping, μW_d, equals the total resistance $(C_{rr}+g)(W_d+W_t)$.**

$$\mu W_d = (C_{rr}+g)(W_d+W_t), \qquad \frac{W_t}{W_d} = \frac{\mu - C_{rr} - g}{C_{rr}+g}$$

Take dry rail $\mu = 0.25$ and wet rail $\mu = 0.10$ (both $C_{rr} = 0.0015$), and rubber tyres $\mu = 0.70$, $C_{rr} = 0.010$.

Question: **can a railway always haul more than rubber tyres?**

2. On the Flat, Rail Wins by Far

Computation 2.1: **for three surfaces, the maximum trailing load was derived at each grade.**

grade	dry rail	wet rail	rubber tyres
0 %	165.666667	65.666667	69.000000
0.5 %	37.461538	14.384615	45.666667
1 %	20.739130	7.695652	34.000000
2.5 %	8.433962	2.773585	19.000000
5 %	3.854369	0.941748	10.666667

> **Observation 2.2**
>
> **on the flat, dry rail hauls 2.400966 times what rubber tyres haul.The limits are 165.666667 against 69.000000, and the flat limit equals $\mu/C_{rr} - 1$——rail's win is the win of a small denominator.**

> **Observation 2.3**
>
> **that win does not come from adhesion.Rubber's adhesion of 0.70 exceeds rail's 0.25, but rail's rolling resistance of 0.0015 is far below rubber's 0.010——losing in the numerator, winning in the denominator.**

3. Grade in the Denominator Erases the Advantage

Computation 3.1: **how many times the denominator (rolling resistance plus grade) grows at a 1 % grade, and the grade at which dry rail and rubber are equal, were derived.**

quantity	value
denominator growth at 1 % (rail)	**7.666667** times
denominator growth at 1 % (rubber)	**2.000000** times
grade at which they are equal	**0.003222** (**0.322222** %)
limit there	**51.941176**

Observation 3.2

this is the core: when grade is added to the same denominator, rail's advantage vanishes.At 1 % rail's denominator grows 7.666667 times while rubber's only doubles, 2.000000, and the limits reverse to 20.739130 against 34.000000——the smaller the denominator, the more the same added grade breaks it.

Observation 3.3

the equal grade is only 0.322222 %.There the limit is 51.941176, less than a third of the flat 165.666667——rail's large margin on the flat is lost to a very slight grade.

Observation 3.4

wet rail loses to rubber even on the flat.On the flat 65.666667 against 69.000000; at 0.5 % the difference from rubber is −31.282051; at 5 % the limit is 0.941748, not even the driving weight itself——when the numerator shrinks, a small denominator cannot make up for it.

4. The Separator

Observation 4.1

the separator is whether the margin is set by the size of adhesion (the numerator) or the smallness of rolling resistance (the denominator).On the flat, rail's small denominator gives a 2.400966 times win; on grades, rubber's large numerator wins——"railways haul heavy loads" does not say which hauls more until the grade is stated.

Observation 4.2

beside Paper 667.Paper 667 showed rolling resistance to be a loss of deformation, not friction, and stated that climbing would not be included——here its smallness sets the load that can be hauled, and adding grade to the same denominator erases the advantage of that smallness.

> **Observation 4.3**
>
> **what is not said.Power, braking, air and curve resistance are not included, coefficients are representative, and why railways avoid grades is not reduced to this——what is shown is that the advantage of a small denominator vanishes on grades.**

5. Roll Call and Closing

1. **dry rail 165.666667** (Computation 2.1): **on the flat.**
2. **rubber tyres 69.000000** (Computation 2.1): **on the flat.**
3. **2.400966 times** (Observation 2.2): **the denominator's win.**
4. **denominator 7.666667 times** (Computation 3.1): **rail breaks down.**
5. **denominator 2.000000 times** (Computation 3.1): **rubber holds up.**
6. **20.739130 against 34.000000 at 1 %** (Observation 3.2): **the core.**
7. **equal at 0.322222 %** (Observation 3.3): **very slight.**
8. **51.941176 there** (Computation 3.1): **under a third of the flat.**
9. **wet rail 65.666667** (Observation 3.4): **losing even on the flat.**
10. **adhesion or rolling resistance** (Observation 4.1): **the separator; Paper 667.**

Closing: **A railway's hauling margin is large only on the flat; once grade enters the denominator, rubber tyres win.Taking the driving weight as 1, on the flat dry rail hauls 165.666667 and rubber tyres 69.000000, rail ahead by 2.400966 times——a win that comes not from adhesion but from the small denominator of rolling resistance.At a 1 % grade the denominator grows 7.666667 times for rail and 2.000000 for rubber, reversing the limits to 20.739130 against 34.000000, and the two are equal at only 0.322222 % (a limit of 51.941176)——the smaller the denominator, the more the same added grade breaks it.Wet rail, at 65.666667, loses to rubber even on the flat.The separator is whether the margin is set by the size of adhesion or the smallness of rolling resistance——"railways haul heavy loads" does not say which hauls more until the grade is stated.Placed among the earlier papers——Paper 667 treated the nature of rolling resistance and left climbing out; this counts how the value of its smallness vanishes on grades.To be honest——power, braking and air resistance are not included, coefficients are representative, and no single cause is claimed.**

References

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