

The Arrow of Time and the Growing Causal Sphere

— Reading Penrose's Weyl Curvature Hypothesis as the Growth of the Light Sphere's Entropy —

The Close of the Thermodynamic Arc

Author: Yuuki YAMAGISHI (山岸 勇輝)

Affiliation: Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Positioning of this paper

This paper offers a **structural interpretation** and is the most ambitious—hence the most carefully limited—of the thermodynamic arc. It does *not* solve the arrow-of-time problem and does not explain the low-entropy past (the "past hypothesis"); Penrose's Weyl Curvature Hypothesis, which it uses, is itself an unproven conjecture about the initial condition. What it does is *read* the gravitational arrow of time geometrically, through the growth of the causal (light) sphere of Paper 61 and its entropy (Papers 65, 73), and place that reading within Penrose's hypothesis and the generalised second law. This closes the emergent-gravity arc (Papers 62, 72, 73, 74): the arrow is the temporal face of gravity-as-thermodynamics. No claim contradicting established theory is made.

ABSTRACT

The second law requires a low-entropy past—the "past hypothesis"—and gravitationally this is subtle: a smooth, uniform distribution is the *low* gravitational-entropy state, and clumping into black holes is the *high* one. Penrose's **Weyl Curvature Hypothesis** encodes this: the initial singularity had vanishing Weyl curvature (smooth, low), final singularities have diverging Weyl (clumped, high); the arrow of time is the growth of gravitational entropy from a smooth beginning. This paper reads that arrow through the system's own geometry. The causal (light) sphere $r = ct$ of Paper 61 grows with time; its area $A = 4\pi(ct)^2$ grows; and its Bekenstein–Hawking entropy $S = A/(4l_P^2)$ (Papers 65, 73) grows accordingly. We record (Proposition 3.1) that the monotone growth of the causal sphere's horizon entropy is a geometric face of the generalised second law, and (Observation 4.1) that it is the expanding-horizon complement of Penrose's clumping Weyl growth. This closes the thermodynamic arc of Papers 62, 72, 73 and 74: under the emergent-gravity umbrella, the arrow of time is gravity-as-thermodynamics read in time. Neither the past hypothesis nor the smallness of the initial Weyl is explained here—these remain Penrose's open conjecture—and the reading is a placement, not a derivation.

1. Introduction

Why does time have a direction? The microscopic laws are essentially time-symmetric, yet the second law makes macroscopic processes irreversible. The deep puzzle, sharpened by Boltzmann and by Penrose, is that the second law requires the universe to have *begun* in a very low-entropy state—the past hypothesis. Gravitationally the low-entropy state is counter-intuitive: a smooth, uniform gas is gravitationally *low* in entropy, and gravitational clumping (ultimately black holes) is *high*. The early universe was hot but smooth—low gravitational entropy—and that is the special initial condition.

Penrose's Weyl Curvature Hypothesis makes this geometric: gravitational entropy lives in the Weyl (tidal, source-free) part of the curvature; the initial singularity had $\text{Weyl} \approx 0$, final singularities have $\text{Weyl} \rightarrow \infty$, and the arrow of time is the growth from smooth to clumped.

This paper reads that arrow through the system's causal sphere—the final panel of the thermodynamic arc (Papers 62, 72, 73, 74): under the emergent-gravity umbrella of Paper 74, the arrow of time is gravity-as-thermodynamics, read in time.

What this paper does not do. It does not explain the past hypothesis or the vanishing initial Weyl (Penrose's own open conjecture), and does not derive the arrow. It reads a known arrow through the system's geometry.

2. The Gravitational Arrow (Established)

Three established facts frame the reading.

- **The past hypothesis** (Boltzmann): the second law's increase holds because the universe began in a very low-entropy macrostate; the arrow is a statement about the initial condition, not the dynamics.
 - **The Weyl Curvature Hypothesis** (Penrose 1979): gravitational entropy is measured by the Weyl curvature $C_{\mu\nu\rho\sigma}$ (the free gravitational field), not the Ricci part (fixed by matter). The initial singularity had $C \rightarrow 0$; final singularities have $C \rightarrow \infty$. Smooth is low, clumped is high.
 - **The generalised second law** (Bekenstein 1974; Gibbons–Hawking 1977): $S_{\text{matter}} + S_{\text{horizon}}$ with $S_{\text{horizon}} = A/(4l_P^2)$ does not decrease; a cosmological (de Sitter) horizon carries such an entropy and grows toward its maximum.
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3. The Causal Sphere Grows

Proposition 3.1 (The causal sphere's entropy grows monotonically)

The causal (light) sphere of Paper 61 is $r = ct$; its area is $A(t) = 4\pi(ct)^2$ and, assigned the Bekenstein–Hawking entropy of Papers 65 and 73,

$$S(t) = \frac{A(t)}{4l_P^2} = \frac{\pi c^2 t^2}{l_P^2}$$

which increases monotonically with t . This growth is a geometric face of the horizon-entropy term in the generalised second law: as the causal horizon expands, the horizon entropy assigned to it increases.

We are careful: the rigorous horizon-entropy statements of the generalised second law are for event and de Sitter horizons; the causal (particle) sphere is the system's geometric handle on the same expanding-horizon phenomenon. The growth of $S(t)$ is the light-sphere image of the arrow, not a new thermodynamic law.

4. Placing It in the Weyl Curvature Hypothesis

Observation 4.1 (Two faces of the gravitational arrow)

The gravitational arrow has two complementary faces, both pointing from the smooth low-entropy beginning:

- **Clumping** (Penrose): matter condenses under gravity, the Weyl curvature grows, and gravitational entropy rises toward black holes—Penrose's Weyl growth.
- **Expansion** (this paper): the causal horizon grows, and the horizon entropy $S(t) = A(t)/4l_P^2$ rises toward its de Sitter maximum—the causal-sphere growth of Paper 61.

The initial state is the common origin: a small causal sphere and near-zero Weyl—low on both counts. The arrow of time is the joint growth. The system's causal sphere is thus the expanding-horizon complement of Penrose's clumping hypothesis; together they are the geometric arrow.

Under the umbrella of Paper 74, this is expected: if gravity is thermodynamics, it has a second law, and the second law has an arrow. Papers 62 and 72 read the cosmological term thermodynamically; Paper 73 read the horizon entropy; Paper 74 named the emergent frame; this paper reads the *time* of that frame.

5. Scope of the Claims

What is established

1. The causal sphere of Paper 61 grows, and its Bekenstein–Hawking entropy $S(t) = \pi c^2 t^2 / l_P^2$ increases monotonically—a geometric face of the growing-horizon term of the generalised second law (Proposition 3.1).
2. This is the expanding-horizon complement of Penrose's Weyl clumping; the two share the smooth low-entropy origin (Observation 4.1). It closes the thermodynamic arc of Papers 62, 72, 73, 74.

What is not established / Open problem

- A solution of the arrow-of-time problem. The past hypothesis—why the universe began at low entropy—is *not* explained.
- An explanation of the vanishing initial Weyl. That is Penrose's conjecture, assumed here, not derived; why the initial condition was so special is left open.
- That the causal-sphere entropy is the whole of the arrow. It is one (horizon) face; the clumping (Weyl) face and the matter entropy are the rest.
- *Open*: does the system's structure constrain the *initial* state—could the solid-angle patch or the $n = 2$ flux geometry make a near-zero initial Weyl natural rather than assumed? That would touch the past hypothesis itself, the deepest question of the arc; it is not attempted here.

6. Conclusion: the Arc Closes

The arrow of time, gravitationally, is the growth of entropy from a smooth low-entropy beginning: Weyl curvature clumping (Penrose) and horizon expansion together. Read through the system's geometry, the expanding side is the growth of the causal sphere of Paper 61 and its Bekenstein–Hawking entropy (Papers 65, 73), $S(t) = \pi c^2 t^2 / l_P^2$ —the light-sphere image of the second law. This is the close of the thermodynamic arc: Papers 62 and 72 read the cosmological term as a boundary/integration constant, Paper 73 read the horizon entropy's coefficient, Paper 74 named the emergent-gravity umbrella, and this paper reads its arrow of time. **All are one stance—gravity as thermodynamics—and the arrow is its**

temporal face. What the arc does not do, and this paper least of all, is explain why the beginning was special; the past hypothesis and the vanishing initial Weyl remain, honestly, Penrose's open conjecture and ours.

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Yuuki YAMAGISHI (山岸 勇輝)

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