How Fast Glass May Be Cooled Falls with the Square of Its Thickness

— passing a 40 K annealing band takes 134.615385 s at 5 mm and 2153.846154 s at 20 mm: four times the thickness gives 0.062500 of the rate and 16.000000 times the time — the separator is whether the material has a yield that caps the stress — [Paper 750]

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— *From thinking to syncing.* —

Abstract

Glass cooled too fast keeps stress inside it. How slowly is slow enough depends strongly on thickness.The allowable cooling rate is counted from the temperature difference and stress within the plate.No new theorem or law is claimed. First, cooling opens a temperature difference between centre and surface——6.250000 K in a 5 mm plate at 1 K/s, with 3.365385 MPa at the surface (Section 2). **Second, thickness sets the allowable rate——0.825397 at 3 mm, 0.297143 at 5 mm, 0.018571 K/s at 20 mm** (Section 3). **Third, and this is the core. The allowable rate goes inversely with thickness squared——from 5 mm to 20 mm the rate becomes 0.062500 of its value** (Section 3). **Fourth, annealing time grows with the square——passing the 40 K band takes 134.615385 s at 5 mm and 2153.846154 s at 20 mm, 16.000000 times** (Section 3). **Fifth, the formula matches a finite-difference solution——after 150 s the difference is 6.250000 K, relative difference 0.000000** (Section 2). **Sixth, the separator is whether the material has a yield that caps the stress——in glass, with no yield, the temperature difference becomes stress directly** (Section 4). **Keywords**: annealing of glass, thermal stress, plate thickness, thermal diffusivity, cooling rate, presence of yield.

Scope of this paper (scope note): **No new theorem or law is claimed——the parabolic temperature profile of a plate cooled at a constant rate, the thermal stress it produces, and annealing rates going inversely with thickness squared are all known (Shelby 2005, Timoshenko and Goodier 1970).Viscoelastic relaxation is not treated——how much stress relaxes within the annealing band is not treated; an allowable stress of 1.0 MPa is used as a set value.The plate is infinitely wide——edges, corners and uneven thickness are not treated.Material constants are representative——modulus, expansion, Poisson's ratio and diffusivity are representative and do not vary with temperature.One annealing band——its width of 40 K is a set value.No real annealing schedule is claimed——particular products and lehrs are not treated.Relation to earlier papers**: **Paper 562 showed that what caps the residual stress in constrained steel as it cools is not the temperature but the yield——there elastic stress far exceeded yield, and varying the temperature difference left the stress stopped at yield. Here glass has no yield to cap it, so the temperature difference becomes stress directly, and the cooling rate and thickness squared that make that difference set the limit. A cap, or none.Paper 352 is also cited——there the cooling rate moved the glass transition temperature; here it makes the temperature difference within the plate. The same "cooling rate" acts on the material in one and on the plate in the other.**What is added is **deriving allowable rates and band-passing times at four thicknesses**, **showing 0.062500 of the rate and 16.000000 times the time for four times the thickness**, **checking the temperature difference against a finite-difference solution**, and **placing the separator on the presence or absence of yield**.

1. Notation and the Question

Setting: **a plate of thickness h is cooled from both faces at a constant rate q. After a while its temperature profile is parabolic, with a difference ΔT between centre and surface, which puts the surface in tension σ_s.**

$$\Delta T = \frac{q\,h^2}{8\kappa}, \qquad \sigma_s = \frac{2}{3}\,\frac{E\alpha}{1-\nu}\,\Delta T, \qquad q_{\max} = \frac{12\,\kappa\,\sigma_{\mathrm{ok}}\,(1-\nu)}{E\alpha\,h^2}$$

Take $E = 70$ GPa, $\alpha = 9.0 \times 10^{-6}$ /K, $\nu = 0.22$, $\kappa = 5.0 \times 10^{-7}$ m²/s and an allowable stress $\sigma_{\mathrm{ok}} = 1.0$ MPa.

Question: **what decides how slowly glass must be cooled?**

2. Cooling Turns a Temperature Difference into Stress

Computation 2.1: **for a 5 mm plate cooled at 1 K/s, the temperature difference and stress were derived by formula and by finite differences.**

quantity	value
centre-to-surface difference (formula)	**6.250000** K
centre-to-surface difference (finite differences, after **150** s)	**6.250000** K
relative difference	**0.000000**
surface stress	**3.365385** MPa

Observation 2.2

at 1 K/s even a 5 mm plate carries 3.365385 MPa at the surface.That exceeds the allowable 1.0 MPa, so the rate is too fast——cooled at the allowable rate, the surface stress is exactly 1.000000 MPa.

3. The Allowable Rate Goes Inversely with Thickness Squared

Computation 3.1: **at four thicknesses, the allowable cooling rate and the time to pass a 40 K annealing band were derived.**

thickness	allowable rate	time through band	same (minutes)
3 mm	**0.825397** K/s	**48.461538** s	**0.807692** min
5 mm	**0.297143** K/s	**134.615385** s	**2.243590** min
10 mm	**0.074286** K/s	**538.461538** s	**8.974359** min
20 mm	**0.018571** K/s	**2153.846154** s	**35.897436** min

Observation 3.2

this is the core: the allowable rate goes inversely with thickness squared.From 5 mm to 20 mm it becomes 0.062500 of its value, differing from 1/16 by 0——the temperature difference is set by the time heat takes to spread through the plate, and that time grows with thickness squared.

Observation 3.3

annealing time grows with thickness squared.Passing the 40 K band takes 134.615385 s at 5 mm and 2153.846154 s at 20 mm, 16.000000 times——four times the thickness acts as sixteen times the time.

4. The Separator

Observation 4.1

the separator is whether the material has a yield that caps the stress.In glass, with no yield, the temperature difference becomes stress directly, and cooling rate and thickness squared set the limit——in a material that yields, stress stops at yield and this proportionality breaks off.

Observation 4.2

beside Paper 562.Paper 562 showed that cooling constrained steel drives elastic stress far beyond yield, and residual stress stops at yield however the temperature difference is varied——there the cap decided the answer; here there is no cap, and the rate and thickness that make the difference decide it.

Observation 4.3

beside Paper 352 as well.Paper 352 showed the glass transition temperature moving with cooling rate——there the rate acts on the material; here on the temperature difference within the plate.

Observation 4.4

what is not said.Viscoelastic relaxation, plate edges and temperature-dependent constants are not treated, and no real annealing schedule is claimed——what is shown is that the allowable rate goes inversely with thickness squared, and that this comes from the absence of yield.

5. Roll Call and Closing

1. **difference 6.250000 K** (Computation 2.1): **1 K/s, 5 mm.**
2. **surface stress 3.365385 MPa** (Computation 2.1): **too fast.**
3. **0.825397 K/s at 3 mm** (Computation 3.1): **a thin plate.**
4. **0.297143 K/s at 5 mm** (Computation 3.1): **the reference.**
5. **0.018571 K/s at 20 mm** (Computation 3.1): **a thick plate.**
6. **0.062500 of the rate** (Observation 3.2): **the core.**
7. **134.615385 s** (Computation 3.1): **annealing at 5 mm.**
8. **2153.846154 s** (Computation 3.1): **annealing at 20 mm.**
9. **16.000000 times** (Observation 3.3): **time grows with the square.**
10. **yield present or absent** (Observation 4.1): **the separator; Paper 562 and Paper 352.**

Closing: **How fast glass may be cooled falls with the square of its thickness.A 5 mm plate cooled at 1 K/s holds 6.250000 K between centre and surface and 3.365385 MPa at the surface——a finite-difference solution also gave 6.250000 K, matching the formula.From an allowable stress of 1.0 MPa, the allowable rate is 0.825397 at 3 mm, 0.297143 at 5 mm and 0.018571 K/s at 20 mm; four times the thickness gives 0.062500 of the rate, and the time through a 40 K band grows from 134.615385 s to 2153.846154 s, 16.000000 times——because the time for heat to spread through the plate grows with thickness squared.The separator is whether the material has a yield that caps the stress——in glass, with no yield, the temperature difference becomes stress directly.Placed among the earlier papers——in Paper 562 yield stopped the stress; here nothing stops it, and rate and thickness set the limit. In Paper 352 the cooling rate acted on the material.To be honest——viscoelastic relaxation and plate edges are not treated, and the allowable stress and band width are set values.**

References

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