Firing Shrinkage Has Two Values Depending on the Denominator, and Volume Shrinkage Is Not Three Times the Linear

— linear shrinkage is **0.150868** on the green size and **0.177674** on the fired size, and a **100** mm fired part needs a **117.767361** mm mould — the separator is whether shrinkage is measured against the green size or the fired size — [Paper 754]

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— From thinking to syncing. —

Abstract

Ceramics shrink on firing, and the shrinkage is often quoted as a single "shrinkage" figure.What that figure is divided by, how volume and length differ, and how density scatter appears in size are counted.No new theorem or law is claimed. First, the same firing has two linear shrinkages——0.150868 on the green size, 0.177674 on the fired size (Section 2). **Second, the mould is sized by the fired denominator——a 100 mm fired part needs a 117.767361 mm mould** (Section 2). **Third, and this is the core. Volume shrinkage is not three times the linear——volume shrinkage 0.387755 against linear shrinkage gives a ratio of 2.570156** (Section 3). **Fourth, three holds only for small strains——2.997001 at linear shrinkage 0.001, 2.440000 at 0.200** (Section 3). **Fifth, size difference appears as a third of the density scatter——a ±1 % green density scatter gives a size difference of 0.006689, 0.334458 of the scatter** (Section 4). **Sixth, the separator is whether shrinkage is measured against the green size or the fired size——"15 % shrinkage" does not fix the mould until the denominator is named** (Section 5). **Keywords**: firing shrinkage, relative density, linear and volume shrinkage, denominator of shrinkage, density scatter, mould size.

Scope of this paper (scope note): **No new theorem or law is claimed——that sintering raises density and shrinks the body, deriving length and volume ratios from mass conservation, expressing shrinkage on either the green or the fired size, and that volume strain equals three times linear strain only as a small-strain approximation are all known (Rahaman 2003, Kingery 1976).The sintering process is not treated——temperature history, sintering rate and grain growth are not treated; only initial and final densities are considered.Shrinkage is isotropic——equal shrinkage in all directions is assumed; anisotropic shrinkage from forming and sagging under weight are not treated.Values are representative——relative densities 0.60 and 0.98 and the ±1 % scatter are set values.No tolerance is claimed——real product tolerances and mould machining accuracy are not treated.Relation to earlier papers**: **Paper 748 showed that most of the water taken out of wood changes neither its size nor its strength——its scope note did not distinguish species or direction and did not touch whether size change is measured in length or volume, or against which denominator. There the water removed and the size change were different numbers. Here the density change and the linear size change are different numbers, and the shrinkage figure itself has two values depending on the denominator.**What is added is **deriving one firing's linear shrinkage on both denominators, 0.150868 and 0.177674**, **deriving the volume-to-linear ratio 2.570156 and showing it leave 3 as strain grows**, **showing that green density scatter appears as roughly a third in size**, and **placing the separator on which size the ruler is set against**.

1. Notation and the Question

Setting: **let green relative density be ρ_g and fired ρ_f. With mass conserved and equal shrinkage in all directions, volume and length ratios follow. Linear shrinkage can be written on the green size L_g or on the fired size L_f.**

$$\frac{V_f}{V_g} = \frac{\rho_g}{\rho_f}, \qquad \frac{L_f}{L_g} = \left(\frac{\rho_g}{\rho_f}\right)^{1/3}, \qquad \varepsilon_g = 1 - \frac{L_f}{L_g}, \qquad \varepsilon_f = \frac{L_g}{L_f} - 1$$

Volume shrinkage is $1 - V_f/V_g = 1 - (1-\varepsilon_g)^3$. Take $\rho_g = 0.60$ and $\rho_f = 0.98$.

Question: **what does a single "shrinkage" figure say?**

2. The Same Firing Has Two Linear Shrinkages

Computation 2.1: **volume and length ratios, linear shrinkage on both denominators and volume shrinkage were derived.**

quantity	value
volume ratio	**0.612245**
length ratio	**0.849132**
linear shrinkage (green denominator)	**0.150868**
linear shrinkage (fired denominator)	**0.177674**
volume shrinkage	**0.387755**
mould for a **100** mm fired part	**117.767361** mm

> **Observation 2.2**
>
> **one firing's linear shrinkage has two values, 0.150868 and 0.177674, by denominator.They convert by $\varepsilon_g = 1 - 1/(1+\varepsilon_f)$, with a difference of 0——both are correct, and the number does not travel unless one says which.**

> **Observation 2.3**
>
> **the mould is sized by the fired denominator.A 100 mm fired part needs a 117.767361 mm mould——sizing the mould from the green-denominator 0.150868 leaves the fired part short.**

3. Volume Shrinkage Is Not Three Times the Linear

Computation 3.1: **for each linear shrinkage (green denominator), the ratio of volume shrinkage to it was derived.**

linear shrinkage	volume shrinkage / linear shrinkage
0.001	2.997001
0.010	2.970100
0.050	2.852500
0.100	2.710000
0.150	2.572500
0.200	2.440000

> **Observation 3.2**
>
> **this is the core: volume shrinkage is not three times the linear.In this firing, volume shrinkage 0.387755 over linear shrinkage 0.150868 is 2.570156——three is $(1-\varepsilon)^3$ cut at first order, and sintering shrinkage is large enough to break that approximation.**

> **Observation 3.3**
>
> **the ratio leaves 3 monotonically as strain grows.2.997001 at 0.001, 2.440000 at 0.200——the "three times" learned from small strains such as thermal expansion cannot be carried into firing.**

4. Size Difference Is a Third of the Density Scatter

Computation 4.1: **with green density scattered by ± 1 % (0.594 and 0.606), the fired size difference from the same mould was derived.**

quantity	value
fired size difference	**0.006689**
difference at **100** mm	**0.668916** mm
ratio to the density scatter **0.02**	**0.334458**

Observation 4.2

size difference appears as roughly a third of the density scatter.Its relative difference from 1/3 is 0.003374——however precisely the mould is made, if green density varies from place to place, a third of that variation becomes size error.

5. The Separator

Observation 5.1

the separator is whether shrinkage is measured against the green size or the fired size.On the green size it is 0.150868, on the fired size 0.177674, and the mould is made from the latter——“15 % shrinkage” does not fix the mould until the denominator is named.

Observation 5.2

beside Paper 748.Paper 748 showed that the water removed from wood and its size change are different numbers——here density change and linear size change are different numbers, and the shrinkage figure itself has two values by denominator.

Observation 5.3

what is not said.The sintering process, anisotropic shrinkage and real tolerances are not treated——what is shown is that one shrinkage becomes two numbers by denominator, and that its ratio to volume is not three.

6. Roll Call and Closing

1. **0.150868 on the green size** (Computation 2.1): **the first value.**
2. **0.177674 on the fired size** (Computation 2.1): **the second value.**
3. **mould 117.767361 mm** (Observation 2.3): **set by the fired denominator.**
4. **volume shrinkage 0.387755** (Computation 2.1): **the volume side.**
5. **ratio 2.570156** (Observation 3.2): **the core.**

6. **2.997001 at 0.001** (Computation 3.1): **three for small strains.**
7. **2.440000 at 0.200** (Computation 3.1): **leaving three.**
8. **size difference 0.006689** (Computation 4.1): **from density scatter.**
9. **0.334458 of the scatter** (Observation 4.2): **a third.**
10. **which size the ruler is set against** (Observation 5.1): **the separator; Paper 748.**

Closing: **Firing shrinkage has two values depending on the denominator, and volume shrinkage is not three times the linear.Shrinking equally in all directions from relative density 0.60 to 0.98, linear shrinkage is 0.150868 on the green size and 0.177674 on the fired size, and a 100 mm fired part needs a 117.767361 mm mould——both are correct, and the number does not travel unless one says which.The ratio of volume shrinkage 0.387755 to linear shrinkage is 2.570156; at 0.001 it is 2.997001 and at 0.200 it is 2.440000, so three holds only for small strains——sintering shrinkage is large enough to break the approximation.A ±1 % green density scatter gives a fired size difference of 0.006689 from the same mould, 0.334458 of the scatter.The separator is whether shrinkage is measured against the green size or the fired size——"15 % shrinkage" does not fix the mould until the denominator is named.Placed among the earlier papers——Paper 748 showed that water removed and size change are different numbers; this counts density and size as different numbers, with shrinkage having two values by denominator.To be honest——the sintering process, anisotropic shrinkage and real tolerances are not treated.**

References

1. Kingery, W. D., Bowen, H. K., & Uhlmann, D. R. (1976). *Introduction to Ceramics* (2nd ed.). Wiley.
2. Rahaman, M. N. (2003). *Ceramic Processing and Sintering* (2nd ed.). Marcel Dekker.
3. Yamagishi, Y. (2026). *Most of the Water Taken Out of Wood Changes Neither Its Size Nor Its Strength* [Paper 748]. Zenodo. 10.5281/zenodo.23015126

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