How a Weld Cools Goes as the Heat Input in a Thick Plate and as Its Square in a Thin One

— the time to cool from 800 to 500 °C is, by the thick-plate formula, 5.101120 s at 2000 J/mm and 20.404480 s at 8000 J/mm — the separator is whether heat escapes in three dimensions or two — [Paper 756]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

Abstract

Weld quality depends strongly on the time the weld takes to cool from 800 to 500 °C. Too fast is bad, and so is too slow.How heat input sets that time is counted for thick and thin plates.No new theorem or law is claimed. First, in a thick plate the cooling time is proportional to heat input——2.550560, 5.101120 and 20.404480 s at 1000, 2000 and 8000 J/mm (Section 2). **Second, in a thin plate it goes as the square——at 10 mm, 9.112118 s at 1000 J/mm and 36.448473 s at 2000 J/mm** (Section 2). **Third, the transition thickness moves with heat input——18.901310 mm at 1000 J/mm, 26.730489 mm at 2000, 53.460977 mm at 8000** (Section 2). **Fourth, and this is the core. The same time window changes the heat-input range with thickness——the range landing in 5 to 20 s spans 4.000000 times in a thick plate and 2.000000 times at 10 mm** (Section 3). **Fifth, the window also shifts far——1960.353816 to 7841.415263 J/mm in a thick plate, 740.756260 to 1481.512519 J/mm at 10 mm** (Section 3). **Sixth, the separator is whether heat escapes in three dimensions or two——"what heat input should be used?" has no answer until the plate thickness is stated** (Section 4). **Keywords**: heat input in welding, cooling time, plate thickness, dimension of heat flow, Rosenthal solution, heat-input window.

Scope of this paper (scope note): No new theorem or law is claimed——the temperature fields of moving point and line sources (Rosenthal's solutions), the thick- and thin-plate formulas for the 800 to 500 °C cooling time, and judging welds by that time are all known (Rosenthal 1946, Easterling 1992).The window is a guide——5 to 20 s is a guide for comparison; no steel-specific or code-specific criterion is claimed.Microstructure is not computed——that too fast gives hard, brittle metal and too slow gives soft, weak metal is only named; hardness and microstructure are not computed.Conditions are stripped——arc efficiency, preheat, joint geometry and penetration shape are not included; heat input is the heat entering the plate.Material constants are fixed——conductivity and heat capacity are representative and independent of temperature.No procedure is recommended——which conditions to weld under is not stated.Relation to earlier papers: Paper 562 showed that what caps a residual stress is not the temperature but the yield——its scope note stated that neither preheat, post-heating nor heat-input control would be discussed. There the upper limit of stress left after cooling was treated, and varying the temperature difference did not move the answer. Here the speed of cooling itself is set by heat input, and the power with which it acts changes from one to two with plate thickness. The limit after cooling, and the speed during it.What is added is tabulating both formulas' times and the transition thickness by heat input, showing that the heat-input range for one window spans **4.000000** times in a thick plate and **2.000000** times at 10 mm, confirming that the two formulas agree at the transition thickness, and placing the separator on three-dimensional against two-dimensional heat flow.

1. Notation and the Question

Setting: let heat input per unit length be q', plate thickness d, conductivity λ, volumetric heat capacity ρc and preheat temperature T_0. In a thick plate heat escapes in three dimensions; in a thin plate, in the two dimensions of its plane. The time to cool from 800 to 500 °C follows the two formulas below, and the thickness at which they agree is the transition d_t.

$$t_{3\mathrm{D}} = \frac{q'}{2\pi\lambda}\left(\frac{1}{500-T_0}-\frac{1}{800-T_0}\right), \qquad t_{2\mathrm{D}} = \frac{(q'/d)^2}{4\pi\lambda\rho c}\left(\frac{1}{(500-T_0)^2}-\frac{1}{(800-T_0)^2}\right)$$

$$d_t = \sqrt{\frac{q'}{2\rho c}\left(\frac{1}{500-T_0}+\frac{1}{800-T_0}\right)}$$

Take $\lambda = 50$ W/(m K), $\rho c = 4.710\times10^{6}$ J/(m^3 K) and $T_0 = 20$ °C.

Question: what heat input puts the cooling time inside a guide window?

2. First Power in a Thick Plate, Second in a Thin One

Computation 2.1: by the thick-plate formula, the cooling time and transition thickness were derived for each heat input.

heat input	cooling time (thick)	transition thickness
1000 J/mm	2.550560 s	18.901310 mm
2000 J/mm	5.101120 s	26.730489 mm
4000 J/mm	10.202240 s	37.802620 mm
8000 J/mm	20.404480 s	53.460977 mm

Computation 2.2: **at a thickness of 10 mm, the thin-plate time was set beside the thick-plate formula.**

heat input	thin-plate formula (10 mm)	thick-plate formula
500 J/mm	2.278030 s	1.275280 s
1000 J/mm	9.112118 s	2.550560 s
2000 J/mm	36.448473 s	5.101120 s

> **Observation 2.3**
>
> **doubling heat input doubles the time in a thick plate and quadruples it in a thin one.In a thin plate heat has one direction fewer to escape, and heat input divided by thickness enters squared——the same change in heat input reaches the time with a different power depending on the plate.**

> **Observation 2.4**
>
> **the transition thickness grows with heat input.At 2000 J/mm it is 26.730489 mm, where the two formulas differ by only 8.882×10^{-16} s——a 10 mm plate is on the thin side at this heat input, taking 36.448473 s rather than the thick formula's 5.101120 s.**

3. The Same Window Changes the Heat-Input Range with Thickness

Computation 3.1: **the heat inputs that land in a 5 to 20 s guide window were solved for by both formulas.**

plate	heat input in the window	span
thick	1960.353816 to 7841.415263 J/mm	4.000000 times
10 mm	740.756260 to 1481.512519 J/mm	2.000000 times

Observation 3.2

this is the core: the same time window spans 4.000000 times in heat input for a thick plate and 2.000000 times at 10 mm.A fourfold time window is fourfold in heat input under the first power, and only its square root, twofold, under the second——the thinner the plate, the narrower the room for setting heat input.

Observation 3.3

the window also shifts far.The thick-plate window starting at 1960.353816 J/mm moves to 740.756260 to 1481.512519 J/mm at 10 mm, never reaching the thick window's lower end——a heat-input figure cannot be reused when the plate changes.

4. The Separator

Observation 4.1

the separator is whether heat escapes in three dimensions or two, that is, whether the plate is thicker or thinner than the transition thickness.In three dimensions cooling time goes as heat input; in two, as its square——"what heat input should be used?" has no answer until the plate thickness is stated.

Observation 4.2

beside Paper 562.Paper 562 showed that yield caps the stress left after cooling, and stated that heat-input control would not be discussed——this counts the speed during cooling and shows the power of heat input changing with thickness. The limit after cooling, and the speed during it.

> **Observation 4.3**
>
> **what is not said.The window is a guide, and arc efficiency, preheat, joint geometry and microstructure are not included——what is shown is that the range and position of heat input for one window change with thickness.**

5. Roll Call and Closing

1. **5.101120 s at 2000 J/mm** (Computation 2.1): **thick plate.**
2. **20.404480 s at 8000 J/mm** (Computation 2.1): **first power.**
3. **transition 26.730489 mm** (Computation 2.1): **moves with heat input.**
4. **36.448473 s at 10 mm** (Computation 2.2): **thin plate.**
5. **four times 9.112118 s** (Observation 2.3): **second power.**
6. **span 4.000000** (Computation 3.1): **thick window.**
7. **span 2.000000** (Computation 3.1): **the core.**
8. **from 1960.353816 J/mm** (Computation 3.1): **thick lower end.**
9. **740.756260 to 1481.512519 J/mm** (Observation 3.3): **thin window.**
10. **three dimensions or two** (Observation 4.1): **the separator; Paper 562.**

Closing: **How a weld cools goes as the heat input in a thick plate and as its square in a thin one.With conductivity 50 W/(m K), a thick plate cools from 800 to 500 °C in 5.101120 s at 2000 J/mm and 20.404480 s at 8000 J/mm, in proportion to heat input——but a 10 mm plate takes 36.448473 s at the same 2000 J/mm, and doubling heat input quadruples the time.The transition thickness is 26.730489 mm at 2000 J/mm and grows with heat input.The heat-input range landing in a 5 to 20 s window narrows from 4.000000 times in a thick plate to 2.000000 times at 10 mm, and shifts from 1960.353816 to 7841.415263 J/mm to 740.756260 to 1481.512519 J/mm——the thinner the plate, the less room, and the same figure cannot be reused.The separator is whether heat escapes in three dimensions or two——"what heat input should be used?" has no answer until the plate thickness is stated.Placed among the earlier papers——Paper 562 treated the stress limit after cooling and set heat-input control aside; this counts the speed during cooling.To be honest——the window is a guide, and arc efficiency, preheat, joint geometry and microstructure are not included.**

References

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Yuuki YAMAGISHI (山岸 勇輝)

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