Which of Three Quantities Buys the Resolution Decides What the Depth of Focus Pays

— halving R leaves depth of focus at 0.500000 if bought with wavelength, 0.250000 with numerical aperture and 1.000000 with k_1 — the separator is whether fineness is bought with wavelength, numerical aperture or k_1 — [Paper 757]

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— From thinking to syncing. —

Abstract

The fineness that exposure can print is often called a limit set by wavelength. But resolution is the product of three quantities: wavelength, numerical aperture and the process factor k_1.What depth of focus pays depending on which of the three bought the fineness is counted.No new theorem or law is claimed. First, features finer than the wavelength can be printed——the representative ArF immersion set gives $R = 42.888889$ nm, 0.222222 of the wavelength (Section 2). **Second, the gain in resolution is a product of three ratios——from g-line to ArF immersion, 0.043036 = wavelength 0.442661 × numerical aperture 0.259259 × k_1 0.375000** (Section 3). **Third, numerical aperture earned the most——logarithmic shares: wavelength 0.259068, numerical aperture 0.429133, k_1 0.311799** (Section 3). **Fourth, and this is the core. The same halving of resolution costs depth of focus in three different ways——0.500000 with wavelength, 0.250000 with numerical aperture, 1.000000 with k_1** (Section 4). **Fifth, so depth of focus shrank faster than resolution——a resolution ratio of 0.043036 against a depth-of-focus ratio of 0.029754** (Section 4). **Sixth, the separator is whether fineness is bought with wavelength, numerical aperture or k_1——"the limit is set by wavelength" does not say what was lost until one says which was used** (Section 5). **Keywords**: resolution limit of exposure, numerical aperture, depth of focus, Rayleigh criterion, immersion lithography, how resolution is bought.

Scope of this paper (scope note): **No new theorem or law is claimed——the Rayleigh forms for resolution as $k_1\lambda/\mathrm{NA}$ and depth of focus proportional to λ/NA^2, numerical aperture above 1 by immersion, and resolution gained by lowering k_1 are all known (Mack 2007, Born and Wolf 1999).The sets are representative——wavelengths, numerical apertures and k_1 of the five sets are representative for comparison; neither real tool values nor a timeline of generations is claimed.Depth of focus is stated only as ratios——computed under the convention $k_2 = 1$; only ratios are claimed, not absolute depth.The content of k_1 is not treated——phase shifting, off-axis illumination and the loss of process margin as k_1 falls are not treated; that buying with k_1 leaves depth of focus unchanged holds on this formula.Process is not treated——pitch splitting is an arithmetic guide; multiple-patterning processes are not treated.Relation to earlier papers**: **Paper 358 showed that the "depth" of depth of field is a convention, not a measurement——its scope note stated that it would not enter the theory of the diffraction limit and computed on geometric optics. This enters the side where diffraction sets resolution and depth of focus, sets the depth coefficient as a convention and claims only ratios. How resolution is bought changes what depth pays.**What is added is **tabulating resolution and depth of focus for five representative sets**, **splitting the resolution gain $\mathbf{0.043036}$ into three ratios with logarithmic shares**, **showing that three ways of halving resolution leave depth of focus at $\mathbf{0.500000}$, $\mathbf{0.250000}$ and $\mathbf{1.000000}$**, and **placing the separator on which of the three buys the fineness**.

1. Notation and the Question

Setting: **let wavelength be λ, numerical aperture NA and the process factor k_1. Printable fineness R and depth of focus DOF take the forms below.**

$$R = k_1 \frac{\lambda}{\mathrm{NA}}, \qquad \mathrm{DOF} = k_2 \frac{\lambda}{\mathrm{NA}^2}, \qquad \frac{R'}{R} = \frac{\lambda'}{\lambda} \cdot \frac{\mathrm{NA}}{\mathrm{NA}'} \cdot \frac{k_1'}{k_1}$$

Take $k_2 = 1$ and consider depth of focus only in ratio.

Question: **what has printable fineness been traded against?**

2. Finer Than the Wavelength

Computation 2.1: **resolution and depth of focus were derived for five representative sets.**

set	wavelength	NA	k_1	resolution R	depth of focus
g-line	436 nm	0.35	0.80	996.571429 nm	3559.183673 nm
i-line	365 nm	0.60	0.60	365.000000 nm	1013.888889 nm
KrF	248 nm	0.80	0.40	124.000000 nm	387.500000 nm
ArF	193 nm	0.93	0.35	72.634409 nm	223.147185 nm
ArF immersion	193 nm	1.35	0.30	42.888889 nm	105.898491 nm

Observation 2.2

the ArF immersion set prints 0.222222 of its wavelength.The g-line set printed 2.285714 times its wavelength——the "wavelength limit" is a limit on the product of three quantities, not on wavelength itself.

3. The Gain Is a Product of Three Ratios

Computation 3.1: **the resolution ratio from g-line to ArF immersion was split into three ratios, with logarithmic shares.**

quantity	ratio	logarithmic share
wavelength	0.442661	0.259068
numerical aperture (inverse)	0.259259	0.429133
k_1	0.375000	0.311799
resolution R	0.043036	1

Observation 3.2

numerical aperture earned the most.The product of the three ratios matches the resolution ratio with a difference of 0, and the shares sum to 1——shortening the wavelength carried only about a quarter of the gain.

4. The Same Halving, Different Costs in Depth

Computation 4.1: **from the ArF immersion set, three ways of halving resolution were taken and the resulting depth-of-focus factor derived.**

way of halving resolution	depth of focus
halve the wavelength	**0.500000** times
double the numerical aperture	**0.250000** times
halve k_1	**1.000000** times

> **Observation 4.2**
>
> **this is the core: the same halving of resolution costs depth of focus in three different ways.Numerical aperture enters resolution at the first power and depth at the second, wavelength enters both at the first, and k_1 does not enter depth——the quantity that earned the most, numerical aperture, pays the most in depth.**

> **Observation 4.3**
>
> **so depth of focus shrank faster than resolution.From g-line to ArF immersion, resolution fell to 0.043036 and depth of focus to 0.029754——having bought much of the fineness with numerical aperture, even more depth was lost.**

> **Observation 4.4**
>
> **pitch splitting is a way outside the three.Splitting the 42.888889 nm pitch of ArF immersion into four is equivalent to 10.722222 nm——it gains fineness without moving the formula's three quantities, but its process is not treated here.**

5. The Separator

Observation 5.1

the separator is whether fineness is bought with wavelength, numerical aperture or k_1.The same halving of resolution returns three depth-of-focus factors, 0.500000, 0.250000 and 1.000000——"the limit is set by wavelength" does not say what was lost until one says which was used.

Observation 5.2

beside Paper 358.Paper 358 showed that the depth of depth of field contains the convention of a "distinguishable limit", and stated that it would not enter the diffraction limit——this is the diffraction side, again setting the depth coefficient k_2 to 1 by convention and claiming only ratios. That depth carries a convention, as Paper 358 said, remains true here.

Observation 5.3

what is not said.The sets are representative; real tools, a timeline of generations, the content of k_1 reduction and multiple-patterning processes are not treated——what is shown is that how resolution is bought splits what depth of focus pays three ways.

6. Roll Call and Closing

1. **ArF immersion 42.888889 nm** (Computation 2.1): **0.222222 of the wavelength.**
2. **resolution ratio 0.043036** (Computation 3.1): **a product of three.**
3. **wavelength 0.442661** (Computation 3.1): **the first ratio.**
4. **numerical aperture 0.259259** (Computation 3.1): **the second ratio.**
5. **share 0.429133** (Observation 3.2): **numerical aperture the largest.**
6. **0.500000 with wavelength** (Computation 4.1): **a first-power cost.**
7. **0.250000 with numerical aperture** (Observation 4.2): **the core.**
8. **1.000000 with k_1** (Computation 4.1): **not in depth.**
9. **depth-of-focus ratio 0.029754** (Observation 4.3): **shrinking faster.**

10. **which of the three buys it** (Observation 5.1): **the separator; Paper 358.**

Closing: **Which of three quantities buys the resolution decides what the depth of focus pays.On representative sets, from g-line at 996.571429 nm to ArF immersion at 42.888889 nm, 0.222222 of the wavelength, the resolution ratio 0.043036 is the product of wavelength 0.442661, numerical aperture 0.259259 and k_1 0.375000, with numerical aperture taking the largest logarithmic share, 0.429133——the "wavelength limit" is a limit on a product of three quantities.To buy the same halving of resolution, depth of focus goes to 0.500000 with wavelength, 0.250000 with numerical aperture and 1.000000 with k_1——numerical aperture, which earned the most, pays the most in depth, and depth of focus fell to 0.029754, below the resolution ratio.The separator is whether fineness is bought with wavelength, numerical aperture or k_1——"the limit is set by wavelength" does not say what was lost until one says which was used.Placed among the earlier papers——Paper 358 showed that depth carries a convention and stayed out of diffraction; this is the diffraction side, with the depth coefficient set by convention and only ratios counted.To be honest——the sets are representative, and real tools, k_1 reduction methods and multiple-patterning processes are not treated.**

References

1. Born, M., & Wolf, E. (1999). *Principles of Optics* (7th ed.). Cambridge University Press.
2. Mack, C. (2007). *Fundamental Principles of Optical Lithography: The Science of Microfabrication*. Wiley.
3. Yamagishi, Y. (2026). *The "Depth" of Depth of Field Is a Convention, Not a Measurement* [Paper 358]. Zenodo. 10.5281/zenodo.22697216

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