In Anisotropic Etching, the Depth of the Pit Is Set by Crystal Planes, Not by Etching Time

— the depth is **71.143691** μm at **100** minutes and **71.576704** μm at **200**, so doubling the time gives **1.006086** times — the separator is whether the rate ratio between the two planes is large — [Paper 758]

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— *From thinking to syncing.* —

Abstract

When a crystal is dissolved to dig a pit, one readily supposes that longer immersion digs deeper. But when the dissolution rate differs between crystal planes, the planes decide the shape.Whether time or crystal planes set the depth is counted, divided by the ratio of plane rates.No new theorem or law is claimed. First, the sidewall angle is set by crystal geometry——(100) and (111) meet at 54.735610 degrees (Section 2). **Second, the pit stops at a depth set by the window——70.710678 μm for a 100 μm window** (Section 2). **Third, and this is the core. Once stopped, time hardly acts——with a rate ratio of 400, going from 100 to 200 minutes deepens the pit only 1.006086 times** (Section 3). **Fourth, with a small rate ratio, time comes back——1.057704 times at ratio 40, 1.573132 times at 4, 2 times isotropically** (Section 3). **Fifth, undercut goes inversely with the rate ratio——0.216506 μm by the stop at ratio 400, 21.650635 μm at 4** (Section 3). **Sixth, the separator is whether the rate ratio between the two planes is large——"to dig deeper, immerse longer" has no truth value until the ratio is stated** (Section 4). **Keywords**: anisotropic etching, crystal planes, sidewall angle, etch-rate ratio, self-limiting depth, undercut.

Scope of this paper (scope note): **No new theorem or law is claimed——anisotropic etching of single-crystal silicon in alkaline solutions, the slowness of (111), its 54.74 degree angle to (100), and self-limiting pyramidal pits are all known (Madou 2002, Seidel et al. 1990).Rates are representative——a (100) rate of 1 µm per minute and ratios of 400, 40 and 4 are for comparison and move with etchant concentration and temperature.The shape is simplified——a square window aligned to the crystal is assumed; convex-corner undercut and mask misalignment are not treated.Sidewalls recede uniformly——the (111) plane is taken to recede along its normal at a constant rate; roughening and the appearance of other planes are not treated.Etchant chemistry is not treated——why (111) is slow is not treated.No real process is claimed——particular tools and products are not treated.Relation to earlier papers**: **Paper 665 showed that the same iron has no direction as a sphere and thirty-eight times the crystal's as a thin film——there, without changing the material, shape created ease of magnetisation. Here, conversely, crystal direction creates the shape of the pit. Shape making direction, and direction making shape.**What is added is **deriving the stopping depth and sidewall angle in closed form**, **tabulating the growth of depth on doubling the time as 1.006086, 1.057704 and 1.573132 times by rate ratio**, **showing undercut inversely proportional to the ratio**, and **placing the separator on whether the rate ratio is large**.

1. Notation and the Question

Setting: **let the (100) rate be R_{100} and the (111) rate $R_{111} = R_{100}/k$. With the angle θ between the planes and a square window of width W, the pit stops at depth h, and the undercut by then is u.**

$$\theta = \arccos\frac{1}{\sqrt{3}}, \qquad h = \frac{W}{2}\tan\theta = \frac{W}{\sqrt{2}}, \qquad u = \frac{R_{111}\,t_h}{\sin\theta}, \quad t_h = \frac{h}{R_{100}}$$

After stopping, the pit deepens only as far as the receding sidewalls widen the effective window. Take $R_{100} = 1$ µm per minute and $W = 100$ µm.

Question: **is the depth of the pit set by immersion time?**

2. Crystal Sets Both Angle and Stopping Depth

Computation 2.1: **the angle between the planes and the stopping depth for a 100 µm window were derived.**

quantity	value
angle between (100) and (111)	**54.735610** degrees
its $\sin$ and $\tan$	**0.816497** and **1.414214**
stopping depth	**70.710678** µm
time to stop	**70.710678** minutes

> **Observation 2.2**
>
> **both the sidewall angle and the stopping depth are set by crystal geometry.The angle equals $\arctan\sqrt{2}$ with a difference of 0, and the stopping depth is $1/\sqrt{2}$ of the window——neither tool settings nor time enter these two numbers.**

3. Once Stopped, Time Hardly Acts

Computation 3.1: **for each rate ratio, the undercut by the stop and the depths at 100 and 200 minutes were derived.**

rate ratio	undercut	depth at 100 min	depth at 200 min	growth
400	**0.216506** µm	**71.143691** µm	**71.576704** µm	**1.006086** times
40	**2.165064** µm	**75.040805** µm	**79.370932** µm	**1.057704** times
4	**21.650635** µm	**100.000000** µm	**157.313218** µm	**1.573132** times

> **Observation 3.2**
>
> **this is the core: with a large rate ratio, time hardly acts once the pit has stopped.At ratio 400, extending from 100 to 200 minutes deepens the pit only 1.006086 times——the pit stops at the depth the window sets, and after that the sidewalls merely recede a little.**

Observation 3.3

with a small rate ratio, time comes back.1.057704 times at ratio 40, 1.573132 times at 4, and isotropically depth doubles with time——the effect of time returns continuously as the ratio approaches 1.

Observation 3.4

undercut goes inversely with the rate ratio.By the stop it is 0.216506 µm at ratio 400 and 2.165064 µm at 40: ten times the ratio, one tenth the undercut——isotropically, digging the same 70.710678 µm undercuts sideways by 70.710678 µm as well.

4. The Separator

Observation 4.1

the separator is whether the rate ratio between the two planes is large: if so, crystal angle sets the shape; near 1, time does.Depth grows 1.006086 times at ratio 400 and 2 times isotropically——“to dig deeper, immerse longer” has no truth value until the ratio is stated.

Observation 4.2

beside Paper 665.Paper 665 showed that shape alone, without changing the material, moves the ease of magnetisation greatly——there shape made direction; here crystal direction makes shape. The two directions of “direction and shape”.

Observation 4.3

what is not said.Convex corners, mask misalignment, etchant chemistry and real processes are not treated ——what is shown is that what sets depth swaps from time to crystal planes with the rate ratio.

5. Roll Call and Closing

1. **angle 54.735610 degrees** (Computation 2.1): **crystal geometry.**
2. **stopping depth 70.710678 µm** (Computation 2.1): **$1/\sqrt{2}$ of the window.**
3. **undercut 0.216506 µm at ratio 400** (Computation 3.1): **slight.**
4. **71.576704 µm at 200 minutes** (Computation 3.1): **hardly moving.**
5. **growth 1.006086 times** (Observation 3.2): **the core.**
6. **1.057704 times at ratio 40** (Observation 3.3): **time begins to return.**
7. **1.573132 times at ratio 4** (Observation 3.3): **time acts.**
8. **undercut 21.650635 µm at ratio 4** (Computation 3.1): **inverse proportion.**
9. **2 times isotropically** (Observation 3.3): **time is everything.**
10. **whether the rate ratio is large** (Observation 4.1): **the separator; Paper 665.**

Closing: **In anisotropic etching, the depth of the pit is set by crystal planes, not by etching time.(100) and (111) meet at 54.735610 degrees, and a 100 µm window gives a pyramidal pit that stops at 70.710678 µm——neither tool settings nor time enter the angle or the depth.With (111) at $1/400$ of the rate, extending from 100 to 200 minutes takes the depth from 71.143691 to 71.576704 µm, only 1.006086 times; at ratio 40 it is 1.057704 times, at 4 it is 1.573132 times, and isotropically 2 times, so time returns as the ratio falls——undercut, too, grows inversely with the ratio, from 0.216506 µm at 400 to 21.650635 µm at 4.The separator is whether the rate ratio between the two planes is large——"to dig deeper, immerse longer" has no truth value until the ratio is stated.Placed among the earlier papers——in Paper 665 shape made direction; here direction makes shape.To be honest——convex corners, mask misalignment and etchant chemistry are not treated, and rates are representative.**

References

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