Amdahl and Gustafson Disagree Only Because the Serial Fraction Is Measured on One Machine or on Many

— putting the serial **0.01** measured on one machine into Amdahl and Gustafson makes them disagree by **99.980201** times on **10000** machines, yet with the share measured on the parallel run the two return the same values — the separator is whether the problem size or the time is held fixed — [Paper 760]

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September 2026

— *From thinking to syncing.* —

Abstract

The limit of parallelisation looks severe under Amdahl's law and generous under Gustafson's. The two are sometimes called opposing laws.The "serial fraction" each takes is counted according to which run it was measured on.No new theorem or law is claimed. First, given the same number, they disagree greatly——with serial 0.01 on 10000 machines, Amdahl 99.019705 times, Gustafson 9900.010000 times, a factor of 99.980201 (Section 2). **Second, serial 0.01 on one machine is most of the parallel run——0.502513 on 100 machines, 0.990197 on 10000** (Section 3). **Third, and this is the core. With the share measured on the parallel run, the two laws agree—— Gustafson returns 50.251256 and 99.019705, the same as Amdahl** (Section 3). **Fourth, the agreement holds across fractions——all 8 cases, parallel fractions 0.99 and 0.90, from 10 to 10000 machines** (Section 3). **Fifth, that share is the value of the same expression as Paper 705's miss share——0.502513 on 100 machines equals the time share of misses at hit rate 0.99 with misses 100 times as costly** (Section 4). **Sixth, the separator is whether the problem size or the time is held fixed——"is the limit 100 times or 9900 times?" has no answer until one says on which run the fraction was measured** (Section 5). **Keywords**: limits of parallelisation, Amdahl's law, Gustafson's law, serial fraction, fixing problem size or time, which run is measured.

Scope of this paper (scope note): **No new theorem or law is claimed——Amdahl's law, Gustafson's law, and that each can be rewritten into the other through how the serial fraction is measured are all known (Amdahl 1967, Gustafson 1988).Work is split in two——only a serial and a parallel part are placed; communication, synchronisation, memory contention and load imbalance are not included.Neither is called right——the two differ only in what they hold fixed, and which question to ask is the user's choice.No real machine is claimed——particular computers and workloads are not treated; 0.99 and 0.90 are values for comparison.Relation to earlier papers**: **Paper 705 showed that even at a hit rate of 0.99, more than half the time is spent on misses——its scope note stated that the derivation of Amdahl's law itself and the limits of parallelisation would not be treated, and that it would only note the same form. This treats that form itself, and shows that the serial share on 100 machines is the value of the same expression as Paper 705's miss share.Paper 702 is also cited——there the exponents of dividing and combining competed, and parallel effects were not included. Here the share of the part that cannot be divided decides, and on which run it is measured changes the answer.**What is added is **putting the disagreement from the same serial 0.01 at 99.980201 times**, **deriving the serial share on the parallel run by machine count**, **confirming that with it the two agree in all 8 cases**, and **placing the separator on holding problem size or time fixed**.

1. Notation and the Question

Setting: **let p be the fraction of one-machine run time that can be parallelised and n the number of machines. Amdahl gives the speed-up with problem size fixed; Gustafson gives the work done with time fixed. Gustafson's serial fraction α is the share of the parallel run's time spent serially.**

$$S_A = \frac{1}{(1-p) + p/n}, \qquad S_G = \alpha + (1-\alpha)\,n, \qquad \alpha = \frac{1-p}{(1-p) + p/n}$$

Putting this α into S_G makes the numerator $(1-p) + p = 1$, so $S_G = S_A$.

Question: **is the limit of parallelisation Amdahl's severe number or Gustafson's generous one?**

2. Given the Same Number, They Disagree Greatly

Computation 2.1: **a serial fraction of 0.01 measured on one machine ($p = 0.99$) was put into both laws as is.**

machines	Amdahl	Gustafson (serial 0.01)
10	9.174312	9.910000
100	50.251256	99.010000
1000	90.991811	990.010000
10000	99.019705	9900.010000

> **Observation 2.2**
>
> given the same number, they disagree by 99.980201 times on 10000 machines.Amdahl sticks under its limit of 100.000000; Gustafson grows almost in proportion to machine count——this is where they look like opposing laws.

3. Measured on the Parallel Run, the Two Agree

Computation 3.1: the serial share α measured on the same parallel run, and Gustafson with that share, were derived.

machines	serial share on the parallel run	Gustafson with it	Amdahl
10	0.091743	9.174312	9.174312
100	0.502513	50.251256	50.251256
1000	0.909918	90.991811	90.991811
10000	0.990197	99.019705	99.019705

> **Observation 3.2**
>
> a serial 0.01 on one machine takes most of the parallel run.0.502513 on 100 machines, 0.990197 on 10000 ——the parallel part shrinks, so the remaining serial part becomes most of the time.

Observation 3.3

this is the core: with the share measured on the parallel run, Gustafson returns the same numbers as Amdahl.With a parallel fraction of 0.90 as well, 5.263158 on 10, 9.174312 on 100, 9.910803 on 1000 and 9.991008 on 10000 machines, all 8 cases agreed——the two are not opposing laws but two rulers applied to the same run.

4. The Same Expression as Paper 705

Observation 4.1

the serial share 0.502513 on 100 machines is the same number as Paper 705's miss share.Paper 705 showed misses taking 0.502513 of the time at hit rate 0.99 with misses 100 times as costly; here the weight is serial 0.01 times the ratio of 100 machines, and both are $1/(1+0.99)$——it is the same point of the "same form" Paper 705 mentioned.

Observation 4.2

with a parallel fraction of 0.90, the share 0.917431 on 100 machines likewise equals Paper 705's miss share at hit rate 0.90.When a small weight meets a large ratio, most of the time moves to it——in caches and in parallelism alike, one expression sets how time is divided.

5. The Separator

Observation 5.1

the separator is whether the problem size or the time is held fixed, which decides on which run the fraction is naturally measured.Fix the problem, and the one-machine share 0.01 is natural, with a limit of 100 times; fix the time, and the parallel-run share is natural, 0.990197 for the same run——"is the limit 100 times or 9900 times?" has no answer until one says on which run the fraction was measured.

Observation 5.2

beside Paper 702.Paper 702 showed the exponents of dividing and combining deciding speed, and left parallel effects out——here the share of the undividable part sets the limit, and how that share is measured swaps the answer.

Observation 5.3

what is not said.Communication, synchronisation and memory contention are not included, and neither law is called right——what is shown is that their disagreement is a difference in which run the fraction is measured on.

6. Roll Call and Closing

1. **Amdahl 99.019705 times** (Computation 2.1): **10000 machines.**
2. **Gustafson 9900.010000 times** (Computation 2.1): **with the same 0.01.**
3. **disagreement 99.980201 times** (Observation 2.2): **looking opposed.**
4. **0.502513 on 100 machines** (Computation 3.1): **serial share on the parallel run.**
5. **0.990197 on 10000** (Computation 3.1): **most of the time.**
6. **50.251256 with it** (Computation 3.1): **equal to Amdahl.**
7. **all 8 cases agree** (Observation 3.3): **the core.**
8. **9.991008 at fraction 0.90 on 10000** (Observation 3.3): **one of the agreements.**
9. **0.917431** (Observation 4.2): **the same expression as Paper 705.**
10. **problem size or time held fixed** (Observation 5.1): **the separator; Paper 705 and Paper 702.**

Closing: **Amdahl and Gustafson disagree only because the serial fraction is measured on one machine or on many.Putting a one-machine serial fraction of 0.01 into both laws as is gives, on 10000 machines, Amdahl 99.019705 times and Gustafson 9900.010000 times, a disagreement of 99.980201 times——they look opposed because the same number was put into both.Measured on the same run from the parallel side, the serial share is 0.502513 on 100 machines and 0.990197 on 10000, and Gustafson with it returns 50.251256 and 99.019705, agreeing with Amdahl in all 8 cases including a parallel fraction of 0.90——the two are two rulers applied to the same run.The share 0.502513 on 100 machines is the value of the same expression as Paper 705's miss share at hit rate 0.99.The separator is whether the problem size or the time is held fixed——"is the limit 100 times or 9900 times?" has no**

answer until one says on which run the fraction was measured.Placed among the earlier papers——Paper 705 only noted the same form, and this treats the form itself; Paper 702 left parallel effects out.To be honest——communication and synchronisation are not included, and neither law is called right.

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