The Five Days of BOD Are a Convention, and One Five-Day Value Hides Different Amounts of Pollution

— Taking decomposition as first order, a rate constant of **0.23** per day makes the five-day value **0.683363** of the ultimate, which reaches **0.99** only after **20.022479** days — the separator is whether the samples compared share the same rate of decomposition — [Paper 762]

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— From thinking to syncing. —

Abstract

Water pollution is often compared by the oxygen microbes use in five days (BOD5), the larger number being read as dirtier.What share of the ultimate oxygen demand the five-day value captures is counted by rate of decomposition.No new theorem or law is claimed. First, the five-day value measures only part of the ultimate——0.683363 at a rate constant of 0.23 per day, reaching 0.99 only after 20.022479 days (Section 2). **Second, the share moves greatly with decomposition rate——0.393469 at 0.10, 0.864665 at 0.40** (Section 2). **Third, and this is the core. The same BOD5 hides different amounts of pollution——at BOD5 200 mg/L the ultimate is 508.298817 or 231.303529 mg/L, 2.197540 times apart** (Section 3). **Fourth, water temperature moves the share too——0.599098 at 15 °C, 0.764696 at 25 °C** (Section 3). **Fifth, the formula matches numerical integration——five-day share 0.683363, differing by 1.887×10^{-15}** (Section 2). **Sixth, the separator is whether the samples compared share the same rate of decomposition——"comparing pollution by BOD5" does not compare amounts until one says the samples share a rate constant** (Section 4). **Keywords**: BOD, biochemical oxygen demand, first-order reaction, rate constant, five-day convention, ultimate BOD.

Scope of this paper (scope note): **No new theorem or law is claimed——the first-order model of BOD, that the five-day value is part of the ultimate, and that the rate constant varies with sample and temperature are all known (Sawyer et al. 2003, Metcalf and Eddy 2003).First order is assumed——lag at the start and departures of real curves from the model are not treated.Nitrification is not treated——oxygen used by oxidising nitrogen is not included.Other indices are not treated——comparison with indices measured by chemical oxidation is not treated.Values are representative——rate constants of 0.10, 0.23 and 0.40 per day and the temperature coefficient 1.047 are for comparison.No standard is claimed——real effluents and regulatory limits are not treated.Relation to earlier papers**: **Paper 358 showed that the depth of depth of field is a convention, not a measurement——there a convention of length, the "distinguishable limit", set the depth, and its choice moved the value. Here a convention of time, "five days", puts different amounts of pollution into the same number. A convention of length, and a convention of time.**What is added is **tabulating the five-day share by rate constant**, **showing the same BOD5 hiding ultimate values 2.197540 times apart**, **deriving how temperature moves the share**, **checking the formula by numerical integration**, and **placing the separator on whether rate constants are the same**.

1. Notation and the Question

Setting: **let the ultimate oxygen demand be L_0 and the rate constant k, with decomposition proceeding at first order. Oxygen used by day t, BOD_t, and the temperature dependence of the rate constant are as follows.**

$$\mathrm{BOD}_t = L_0\left(1 - e^{-kt}\right), \qquad \frac{\mathrm{BOD}_5}{L_0} = 1 - e^{-5k}, \qquad k_T = k_{20}\,\theta^{\,T-20}$$

Take a reference rate constant of 0.23 per day and a temperature coefficient $\theta = 1.047$.

Question: **are two waters with the same BOD5 equally polluted?**

2. The Five-Day Value Measures Only Part

Computation 2.1: **for each rate constant, the share of the ultimate captured by the five-day value was derived.**

rate constant	five-day share
0.10 per day	**0.393469**
0.23 per day	**0.683363**
0.40 per day	**0.864665**

Observation 2.2

the five-day value measures only part of the ultimate.At 0.23 per day it is 0.683363, and reaching 0.99 takes 20.022479 days——stopping at five days is not a natural break but a convention.

Observation 2.3

the share moves greatly with decomposition rate.0.393469 at 0.10, 0.864665 at 0.40——the same five days capture a different share for each sample.

Observation 2.4

the formula matches numerical integration.Integrating the decay in small steps gave a five-day share of 0.683363, differing from the formula by 1.887×10^{-15}——the difference in share comes from the rate constant, not from how the model is written.

3. The Same BOD5 Hides Different Pollution

Computation 3.1: **for a common BOD5 of 200 mg/L, the ultimate value was derived for each rate constant.**

rate constant	ultimate value
0.10 per day	**508.298817** mg/L
0.23 per day	**292.670122** mg/L
0.40 per day	**231.303529** mg/L

Observation 3.2

this is the core: the same BOD5 hides different amounts of pollution.At the same 200 mg/L, the ultimate is 508.298817 or 231.303529 mg/L, 2.197540 times apart——the more slowly pollution decomposes, the smaller the five-day value makes it look.

Computation 3.3: **with the rate constant changing with water temperature, the five-day share was derived.**

water temperature	rate constant	five-day share
15 °C	0.182808 per day	0.599098
20 °C	0.230000 per day	0.683363
25 °C	0.289375 per day	0.764696

Observation 3.4

even for one sample, temperature moves the share.0.599098 at 15 °C, 0.764696 at 25 °C——so unless the measuring temperature is also fixed by convention, five-day values cannot be compared.

4. The Separator

Observation 4.1

the separator is whether the samples compared share the same rate of decomposition (rate constant).If they do, the ratio of BOD5 equals the ratio of ultimate values; if not, the same BOD5 hides a factor of 2.197540 ——"comparing pollution by BOD5" does not compare amounts until one says the samples share a rate constant.

Observation 4.2

beside Paper 358.Paper 358 showed that the depth of depth of field contains a convention of length, the "distinguishable limit"——here a convention of time, "five days", puts different amounts of pollution into the same number. In both, the convention is not wrong, but comparing numbers without stating it misleads.

Observation 4.3

what is not said.Lag, nitrification, other indices and real standards are not treated——what is shown is that the share captured in five days moves with rate constant and temperature, and the same number hides different pollution.

5. Roll Call and Closing

1. **0.683363 in five days** (Computation 2.1): **part of the ultimate.**
2. **20.022479 days to 0.99** (Observation 2.2): **five days is a convention.**
3. **0.393469 at 0.10** (Computation 2.1): **slow decomposition.**
4. **0.864665 at 0.40** (Computation 2.1): **fast decomposition.**
5. **ultimate 508.298817 mg/L** (Computation 3.1): **slow pollution.**
6. **231.303529 mg/L** (Computation 3.1): **fast pollution.**
7. **2.197540 times** (Observation 3.2): **the core.**
8. **0.599098 at 15 °C** (Computation 3.3): **moved by temperature.**
9. **integration 0.683363** (Observation 2.4): **matching the formula.**
10. **same rate constant or not** (Observation 4.1): **the separator; Paper 358.**

Closing: **The five days of BOD are a convention, and one five-day value hides different amounts of pollution.With first-order decomposition at 0.23 per day, the five-day value captures only 0.683363 of the ultimate, which reaches 0.99 after 20.022479 days——stopping at five days is not a natural break.At rate constants of 0.10 and 0.40 the five-day share is 0.393469 and 0.864665, so the same BOD5 of 200 mg/L means ultimate values of 508.298817 and 231.303529 mg/L, 2.197540 times apart——the more slowly pollution decomposes, the smaller the five-day value makes it look.Even one sample's share moves with temperature, 0.599098 at 15 °C and 0.764696 at 25 °C.The separator is whether the samples compared share the same rate of decomposition——"comparing pollution by BOD5" does not compare amounts until one says the samples share a rate constant.Placed among the earlier papers——Paper 358 showed a convention of length; this counts a convention of time putting different amounts into the same number.To be honest——first order is assumed, and lag, nitrification, other indices and real standards are not treated.**

References

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