Contact Angle, Not Density, Decides Float or Sink, and at Some Sizes the Heavier Mineral Floats and the Lighter Sinks

— for particles between **0.225167** and **0.836686** mm, galena, with **3.939394** times the density excess over water, floats and quartz sinks — the separator is whether surface wetting that holds a particle to a bubble beats its weight in water — [Paper 764]

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— From thinking to syncing. —

Abstract

Crushed ore is put in water and separated by attaching particles to bubbles. Heavy things should sink, yet the heavier mineral can be the one floated.Whether density or contact angle decides float or sink is counted by balancing the holding force against weight.No new theorem or law is claimed. First, the gain of bubble attachment grows with contact angle——$\gamma(1-\cos\theta)$ is 1.093842 at 10 degrees and 59.497331 mN/m at 80 (Section 2). **Second, the factor of the holding force is $\sin^2(\theta/2)$——0.007596 at 10 degrees, 0.413176 at 80** (Section 2). **Third, floatable particle size has an upper limit——galena at 80 degrees up to 0.836686 mm, quartz at 10 degrees up to 0.225167 mm** (Section 3). **Fourth, and this is the core. There is a size at which the heavier mineral floats and the lighter sinks——between 0.225167 and 0.836686 mm, galena with 3.939394 times the density excess floats and quartz sinks** (Section 3). **Fifth, at equal contact angle, density acts——floatable radius goes inversely with the square root of density excess; galena to quartz is 0.503831** (Section 3). **Sixth, the separator is whether surface wetting that holds a particle to a bubble beats its weight in water——"heavy things sink" does not say which floats until the wetting is stated** (Section 4). **Keywords**: froth flotation, contact angle, collector, holding force on a bubble, floatable particle size, density and wetting.

Scope of this paper (scope note): **No new theorem or law is claimed——that flotation separates minerals by surface wetting, that collectors raise the contact angle, that the maximum capillary force holding a sphere to a bubble is approximated by $2\pi R\gamma\sin^2(\theta/2)$, and that floatable particle size has an upper limit are all known (Nguyen and Schulze 2004, Fuerstenau et al. 2007).Only static balance——time to attach, detachment by turbulence, bubble size and particle shape are not treated.The force is an approximation——the maximum capillary force for a sphere at a flat interface is used; bubble curvature and buoyancy details are not included.Values are representative——densities 7500, 2650 and 1300 and contact angles 10, 40 and 80 degrees are for comparison.No real operation is claimed——particular mines and reagents are not treated.Relation to earlier papers**: **Paper 309 showed that the length dividing gravity from surface tension is nobody's size——its scope note stated that contact angle would not be treated, computing with $\cos\theta = 1$ (complete wetting). Here contact angle cuts the effect of surface tension by $\sin^2(\theta/2)$, and the upper limit of floatable size is set by contact angle and density excess together.Paper 722 is also cited——there contact angle flipped the sign of capillary pressure. Here it sets the size of the force holding a particle to a bubble.**What is added is **setting the gain of bubble attachment beside the work of water adhesion and showing them opposite**, **confirming the maximum force factor on a grid**, **deriving floatable radii for three minerals**, **showing a size window where the heavier mineral floats and the lighter sinks**, and **placing the separator on wetting against weight**.

1. Notation and the Question

Setting: **let surface tension be γ and contact angle θ. The work of water adhering to a solid and the gain of a bubble displacing water to attach are the two below. The radius at which the maximum force holding a sphere of radius R to a bubble balances its weight in water is the upper limit $R_{\max}$ of floatable size.**

$$W_{\text{water}} = \gamma(1+\cos\theta), \qquad W_{\text{bubble}} = \gamma(1-\cos\theta), \qquad 2\pi R\gamma\sin^2\frac{\theta}{2} = (\rho_p-\rho_w)\,g\,\frac{4}{3}\pi R^3$$

$$R_{\max} = \sqrt{\frac{3\gamma\sin^2(\theta/2)}{2(\rho_p-\rho_w)g}}$$

Take $\gamma = 0.072$ N/m.

Question: **is float or sink decided by how heavy the mineral is?**

2. The Gain of Attachment and the Holding Force

Computation 2.1: **for each contact angle, the work of water adhesion, the gain of bubble attachment and the force factor were derived.**

contact angle	water adhesion	bubble attachment	force factor $\sin^2(\theta/2)$
10 degrees	142.906158 mN/m	1.093842 mN/m	0.007596
40 degrees	127.155200 mN/m	16.844800 mN/m	0.116978
60 degrees	108.000000 mN/m	36.000000 mN/m	0.250000
80 degrees	84.502669 mN/m	59.497331 mN/m	0.413176
90 degrees	72.000000 mN/m	72.000000 mN/m	0.500000

Observation 2.2

the gain of bubble attachment grows with contact angle, and the work of water adhesion moves the opposite way.At 80 degrees the bubble gain is 59.497331 mN/m and water adhesion 84.502669 mN/m——the number pointing toward flotation is the bubble gain, not water adhesion.

Observation 2.3

the holding-force factor is $\sin^2(\theta/2)$.Moving the contact line over a grid of angles to find the maximum matched that value with a difference of 0 at all three angles——at 10 degrees only 0.007596 acts; at 80 degrees, 0.413176.

3. A Size at Which the Heavier Mineral Floats

Computation 3.1: **for three minerals, the largest floatable radius was derived at each contact angle.**

mineral (density)	10 degrees	40 degrees	80 degrees
galena (7500)	0.113446 mm	0.445191 mm	0.836686 mm
quartz (2650)	0.225167 mm	0.883611 mm	1.660646 mm
coal (1300)	0.528064 mm	2.072252 mm	3.894560 mm

Observation 3.2

at equal contact angle, the heavier the mineral, the smaller the particles that float.Floatable radius goes inversely with the square root of density excess, galena to quartz being 0.503831 at 40 degrees——with the same surface, they sink in order of density.

Observation 3.3

this is the core: there is a size at which the heavier mineral floats and the lighter sinks.Galena raised to 80 degrees by a collector floats up to 0.836686 mm, while quartz left at 10 degrees floats only up to 0.225167 mm. Between them, galena with 3.939394 times the density excess over water floats and quartz sinks——the collector does not change the mineral; it changes only the contact angle, and reorders flotation away from density.

4. The Separator

Observation 4.1

the separator is whether surface wetting that holds a particle to a bubble beats its weight in water.The wetting factor $\sin^2(\theta/2)$ moves from 0.007596 to 0.413176, more than fifty-fold, and can overcome a density excess of 3.939394 times——“heavy things sink” does not say which floats until the wetting is stated.

Observation 4.2

beside Paper 309.Paper 309 derived the length dividing gravity from surface tension under complete wetting and stated that contact angle would not be treated——here contact angle cuts the effect of surface tension, and floatable size is set by contact angle and density excess together.

Observation 4.3

beside Paper 722 as well.Paper 722 showed contact angle flipping the sign of capillary pressure——here contact angle sets the size of the force holding a particle to a bubble.

Observation 4.4

what is not said.Attachment time, turbulence, bubble size and real operations are not treated, and the force is an approximation——what is shown is that there are sizes where contact angle, not density, sets the order of flotation.

5. Roll Call and Closing

1. **bubble gain 59.497331 mN/m** (Computation 2.1): **80 degrees.**
2. **water adhesion 84.502669 mN/m** (Computation 2.1): **the opposite way.**
3. **force factor 0.007596** (Computation 2.1): **10 degrees.**
4. **0.413176** (Computation 2.1): **80 degrees.**
5. **galena 0.836686 mm** (Computation 3.1): **floats at 80 degrees.**
6. **quartz 0.225167 mm** (Computation 3.1): **limit at 10 degrees.**
7. **density excess 3.939394 times** (Observation 3.3): **the heavier floats.**
8. **ratio 0.503831** (Observation 3.2): **density order at equal angle.**
9. **window 0.225167 to 0.836686 mm** (Observation 3.3): **the core.**
10. **wetting against weight** (Observation 4.1): **the separator; Paper 309 and Paper 722.**

Closing: **What decides float or sink is the contact angle, not the density, and there is a size at which the heavier mineral floats and the lighter sinks.The gain of bubble attachment $\gamma(1-\cos\theta)$ grows with contact angle to 59.497331 mN/m at 80 degrees, and the holding-force factor $\sin^2(\theta/2)$ runs from 0.007596 at 10 degrees to 0.413176 at 80——at equal contact angle, floatable radius goes inversely with the square root of density excess, and minerals sink in order of density.But galena raised to 80 degrees floats up to 0.836686 mm while quartz at 10 degrees floats only up to 0.225167 mm, so between them galena, with 3.939394 times the density excess, floats and quartz sinks ——the collector reorders flotation by changing only the contact angle.The separator is whether surface wetting that holds a particle to a bubble beats its weight in water——"heavy things sink" does not say which floats until the wetting is stated.Placed among the earlier papers——Paper 309 derived its length under complete wetting and left contact angle aside; Paper 722 had contact angle flip a sign; this counts contact angle setting floatable size.To be honest——attachment time, turbulence and bubble size are not treated, and the force is an approximation.**

References

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