When There Are Many Shortest Paths, What Picks One Is Not the Optimisation but How the Tie Is Broken

— On a 10×10 grid with all weights 1, there are 184756 shortest paths of length 20 — the separator is whether the weights allow ties, as integers do, or rule them out, as real numbers do — [Paper 768]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

A shortest-path computation returns one answer. But when many paths are equally short, that one was not picked by the computation.The number of ties, and the variety of paths picked by perturbations that break them, are counted.No new theorem or law is claimed. First, a grid has a vast number of shortest paths——184756 paths of length 20 on 10×10, all equally long (Section 2). **Second, a small perturbation makes the shortest path unique——adding random amounts of 10^{-9} gave exactly 1 shortest path all 1000 times** (Section 2). **Third, and this is the core. The unique path changes with how the perturbation is added——1000 ways of adding it selected 998 different paths** (Section 2). **Fourth, a different rule changes the path entirely——"right first" and "up first" share 0 edges** (Section 3). **Fifth, ties arise with integer weights, not with real ones——several shortest paths in a share 0.973000 of instances with weights in $\{1, 2\}$, and 0.000000 with real weights in $[1, 2]$** (Section 3). **Sixth, the separator is whether the weights allow ties, as integers do, or rule them out, as real numbers do——"the optimum was found" does not say whether that one path means anything until one says whether ties exist** (Section 4). **Keywords**: shortest path, non-uniqueness of paths, tie-breaking, binomial coefficient, perturbation, grid paths.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——that the number of monotone grid paths is a binomial coefficient, finding shortest paths by dynamic programming, and breaking ties by small perturbations are all known (Dijkstra 1959, Cormen et al. 2009).Grids only——only grid paths moving right or up are considered; general graphs are not treated.Implementation is not treated——real maps and route-finding systems are not treated.One kind of perturbation——perturbations are uniform random numbers in $[0, 10^{-9}]$.No rule is recommended——which rule should break ties is not stated.Relation to earlier papers**: **Paper 340 showed that the shorter path is not the one a navigator steers——there the criterion of holding a constant bearing made the navigator choose a path that is not shortest. Here 184756 shortest paths stand side by side, and which one is picked is decided not by the optimisation but by how perturbations are added. A criterion that abandons the shortest, and one that chooses among the shortest.**What is added is **recounting the number of shortest paths**, **showing that 1000 perturbations of 10^{-9} select 998 different paths**, **showing that two tie-breaking rules share no edge**, **setting the share of ties with integer and real weights side by side**, and **placing the separator on whether the weights allow ties**.

1. Notation and the Question

Setting: **on a 10×10 grid, move from lower left to upper right, only right or up. Put weights on edges; the path of least total weight is the shortest path. With all weights 1, every path has length 20, and the number of them is as follows.**

$$N = \binom{20}{10}, \qquad P(\text{passing the centre}) = \frac{\binom{10}{5}^2}{\binom{20}{10}}$$

The number of shortest paths is counted by dynamic programming, summing over ties.

Question: **what picked the one path a shortest-path computation returns?**

2. A Perturbation That Breaks Ties Picks One

Computation 2.1: **shortest paths were counted with all weights 1, and then the grid was solved 1000 times with random amounts of 10^{-9} added to the weights.**

quantity	value
shortest length	**20**
number of shortest paths	**184756**
always exactly **1** shortest path with perturbation	yes (all **1000** times)
different paths selected	**998**

> **Observation 2.2**
>
> **a grid has 184756 shortest paths, all of length 20.The count by dynamic programming matched the binomial coefficient——the optimum is not a point but a large set.**

> **Observation 2.3**
>
> **this is the core: a perturbation makes the shortest path unique, but that path changes each time with how the perturbation is added.1000 ways of adding it selected 998 different paths——what picks "the unique optimum" is not the cost but a perturbation of 10^{-9}.**

3. Rules and Kinds of Weight

Computation 3.1: **the paths of two tie-breaking rules were compared, the probability that a uniformly chosen shortest path passes the centre was found by formula and trial, and the share of ties was counted over 1000 instances with integer and with real weights.**

quantity	value
edges shared by "right first" and "up first"	**0**
probability of passing the centre (formula)	**0.343718**
probability of passing the centre (trial, **40000** paths)	**0.345175**
share with several shortest paths, weights $\{1, 2\}$	**0.973000**
share with several shortest paths, real weights $[1, 2]$	**0.000000**

Observation 3.2

a different rule changes the path entirely.“Right first” and “up first” choose paths of the same length 20 that share no edge——how ties are broken is not a detail; it decides the whole answer.

Observation 3.3

chosen uniformly, paths crowd toward the middle of the grid.The probability of passing the centre is 0.343718 by formula and 0.345175 by trial——change the way of choosing, and the distribution of where paths go changes too.

Observation 3.4

ties arise with integer weights, not with real ones.With weights in $\{1, 2\}$ a share 0.973000 of instances has several shortest paths; with real weights in $[1, 2]$, 0.000000——once distances or costs are rounded to integers, ties arise almost always.

4. The Separator

Observation 4.1

the separator is whether the weights allow ties, as integers do, or rule them out, as real numbers do.When ties are allowed, the one path an optimiser returns is the one a perturbation or a rule picked——“the optimum was found” does not say whether that one path means anything until one says whether ties exist.

Observation 4.2

beside Paper 340.Paper 340 showed that a criterion of holding a constant bearing chooses a path that is not shortest——there a criterion abandoned the shortest; here a perturbation picks one from among the shortest.

Observation 4.3

what is not said.General graphs, real maps and which rule should break ties are not treated——what is shown is that in an optimisation with ties, what chooses is not the cost.

5. Roll Call and Closing

1. **184756 shortest paths** (Computation 2.1): **all of length 20.**
2. **1 path with perturbation** (Computation 2.1): **all 1000 times.**
3. **998 different paths selected** (Observation 2.3): **the core.**
4. **0 shared edges** (Observation 3.2): **a rule changes the whole path.**
5. **centre 0.343718** (Computation 3.1): **formula.**
6. **trial 0.345175** (Computation 3.1): **on the formula.**
7. **integer weights 0.973000** (Observation 3.4): **ties almost always.**
8. **real weights 0.000000** (Observation 3.4): **no ties.**
9. **a perturbation of 10^{-9}** (Observation 2.3): **what chooses, instead of cost.**
10. **ties allowed or not** (Observation 4.1): **the separator; Paper 340.**

Closing: **When there are many shortest paths, what picks one is not the optimisation but how the tie is broken.On a 10×10 grid with all weights 1 there are 184756 shortest paths of length 20——the optimum is not a point but a large set.Adding random amounts of 10^{-9} always leaves exactly 1 shortest path, yet 1000 ways of adding them select 998 different paths, and “right first” and “up first” share 0 edges——what picks “the unique optimum” is not the cost but the perturbation and the rule.With integer weights in $\{1, 2\}$ a share 0.973000 of instances has several shortest paths, with real weights 0.000000: rounding makes ties almost certain.The separator is whether the weights allow ties, as integers do, or rule them out, as real numbers do——“the optimum was found” does not say whether that one path means anything until one says whether ties exist.Placed among the earlier papers——Paper 340 showed a criterion abandoning the shortest; this counts a perturbation choosing among the shortest.To be honest——only monotone grid paths are considered, not general graphs or real maps.**

References

1. Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (3rd ed.). MIT Press.
2. Dijkstra, E. W. (1959). A note on two problems in connexion with graphs. *Numerische Mathematik*.

3. Yamagishi, Y. (2026). *The Shorter Path Is Not the One a Navigator Steers* [Paper 340]. Zenodo. 10.5281/zenodo.22682089

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)