On a Round Trip, Optical Rotation Undoes Itself and Faraday Rotation Doubles, and Only the Latter Can Make Light One-Way

— a medium turning polarisation by **45** degrees each way returns to **0.000000** degrees after a round trip with optical activity and reaches **90.000000** with Faraday rotation — the separator is whether the sense of rotation is set by the material's handedness or by an applied magnetic field — [Paper 773]

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— *From thinking to syncing.* —

Abstract

Elements that turn polarisation include optically active materials, which turn it themselves, and Faraday rotators, which turn it in a magnetic field. Seen one way, both turn light alike.The rotation after going and returning, and the ability to pass light in one direction only, are counted.No new theorem or law is claimed. First, on a round trip, optical rotation undoes itself——45 degrees each way gives 0.000000 degrees round trip (Section 2). **Second, on a round trip, Faraday rotation doubles——45 degrees each way gives 90.000000 degrees round trip** (Section 2). **Third, and this is the core. Only Faraday rotation can make light one-way——between polarisers, backward transmission is 0 with Faraday rotation and 1.000000 with optical activity** (Section 3). **Fourth, rotation error becomes backward leakage——a 1 degree error leaks 3.045865×10^{-4}, an isolation of 35.162894 dB** (Section 3). **Fifth, the length needed is set by field and Verdet constant——5.861180 mm for 45 degrees at 134 rad/(T m) and 1 T** (Section 3). **Sixth, the separator is whether the sense of rotation is set by the material's handedness relative to the direction of travel or by the direction of an applied magnetic field——"an element that turns light by 45 degrees" does not say whether it can stop light until one says which** (Section 4). **Keywords**: polarisation rotation, optical activity, Faraday rotation, optical isolator, reciprocal and non-reciprocal, Jones matrix.

Scope of this paper (scope note): **No new theorem or law is claimed——optical activity and Faraday rotation, polarisation calculations with Jones matrices, cancellation of rotation on a round trip through a reciprocal medium, and the use of Faraday rotation in optical isolators are all known (Hecht 2017, Saleh and Teich 2007).Losses are not treated——absorption, reflection losses and imperfect polarisers are not included.One wavelength——the wavelength dependence of rotation is not treated.Values are representative——the Verdet constant 134 rad/(T m) is for comparison.Other effects are not treated——polarisation changes by the electro-optic effect are only named.No real device is claimed——the performance of particular isolators is not treated.Relation to earlier papers**: **Paper 335 showed that the round trip changes the exponent——there a radar round trip changed the power of distance from 2 to 4. Here a round trip makes polarisation rotation cancel in a reciprocal medium and add in a non-reciprocal one. A round trip changing an exponent, and a round trip deciding whether to cancel or to add.**What is added is **computing round-trip rotation in two media**, **showing backward transmission between polarisers splitting into 0 and 1.000000**, **tabulating rotation error against isolation depth**, and **placing the separator on handedness against magnetic field**.

1. Notation and the Question

Setting: **represent polarisation by a two-component vector and rotation by the matrix $R(\theta)$. When light that received $R(\theta)$ going forward returns, seen in laboratory coordinates, a reciprocal medium (optical activity) turns it the opposite way, and a non-reciprocal medium (Faraday rotation) turns it the same way.**

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \qquad M_{\text{recip}} = R(-\theta)R(\theta) = I, \qquad M_{\text{nonrecip}} = R(\theta)R(\theta) = R(2\theta)$$

The Faraday rotation angle is $\theta = VBL$, with V the Verdet constant, B the field and L the length.

Question: **are two elements that turn light equally one way also equal on a round trip?**

2. Round-Trip Rotation

Computation 2.1: **the round-trip rotation through a medium turning 45 degrees each way was derived for the two media.**

medium	round-trip rotation
optical activity (reciprocal)	0.000000 degrees
Faraday rotation (non-reciprocal)	90.000000 degrees

Observation 2.2

on a round trip, optical rotation undoes itself and Faraday rotation doubles.The reciprocal round trip matched the identity to 2.220×10^{-16}, and the non-reciprocal one matched a 90 degree rotation to the same ——two elements indistinguishable one way become opposites on a round trip.

3. Making Light One-Way

Computation 3.1: **light was passed through a polariser at 0 degrees, a 45 degree rotator and an analyser at 45 degrees, forward and backward. Backward leakage was also derived for Faraday rotation departing from 45 degrees.**

quantity	Faraday rotation	optical activity
forward transmission	1.000000	1.000000
backward transmission	0	1.000000

rotation error	backward leakage	isolation
0.5 degrees	7.615242×10^{-5}	41.183163 dB
1.0 degrees	3.045865×10^{-4}	35.162894 dB
2.0 degrees	1.217975×10^{-3}	29.143617 dB
5.0 degrees	7.596123×10^{-3}	21.194080 dB

Observation 3.2

this is the core: only Faraday rotation can make light one-way.Returning light is turned a further 45 degrees to 90 by Faraday rotation and stopped by the polariser, but turned back 45 degrees to 0 by optical activity and passed at 1.000000——one-way passage needs different behaviour going and returning, and only a non-reciprocal medium has it.

Observation 3.3

rotation error becomes backward leakage.The leakage equals the $\sin^2$ of the error: 35.162894 dB at 1 degree, down to 21.194080 dB at 5 degrees——the precision of setting 45 degrees decides the depth of isolation.

Observation 3.4

the length needed is set by field and Verdet constant.With a Verdet constant of 134 rad/(T m) and a field of 1 T, 45 degrees takes 5.861180 mm——the sense of rotation is set not by length or material but by the direction of the field.

4. The Separator

Observation 4.1

the separator is whether the sense of rotation is set by the material's handedness relative to the direction of travel or by the direction of an applied magnetic field.With handedness the sense reverses with the direction of travel and cancels on a round trip; with a field it is fixed and adds——"an element that turns light by 45 degrees" does not say whether it can stop light until one says which.

Observation 4.2

beside Paper 335.Paper 335 showed a radar round trip changing the power of distance from 2 to 4——here a round trip cancels in a reciprocal medium and adds in a non-reciprocal one. What a round trip does is decided by whether going and returning follow the same rule.

Observation 4.3

what is not said.Losses, wavelength dependence, the electro-optic effect and real device performance are not treated——what is shown is that cancelling or adding on a round trip decides whether one-way passage can be built.

5. Roll Call and Closing

1. **optical activity round trip 0.000000 degrees** (Computation 2.1): **undone.**
2. **Faraday round trip 90.000000 degrees** (Computation 2.1): **doubled.**
3. **forward transmission 1.000000** (Computation 3.1): **both pass.**
4. **backward transmission 0** (Observation 3.2): **Faraday stops it.**
5. **optical activity backward 1.000000** (Observation 3.2): **the core.**
6. **35.162894 dB at 1 degree error** (Observation 3.3): **precision is depth.**
7. **21.194080 dB at 5 degrees** (Computation 3.1): **shallower.**
8. **41.183163 dB at 0.5 degrees** (Computation 3.1): **deeper.**
9. **5.861180 mm for 45 degrees** (Observation 3.4): **at 1 T.**
10. **handedness or field** (Observation 4.1): **the separator; Paper 335.**

Closing: **On a round trip, optical rotation undoes itself and Faraday rotation doubles, and only the latter can make light one-way.A medium turning polarisation 45 degrees each way returns to 0.000000 degrees round trip with optical activity and reaches 90.000000 degrees with Faraday rotation——indistinguishable one way, they become opposites on a round trip.Between a polariser at 0 degrees and an analyser at 45, forward transmission is 1.000000 for both, but backward transmission is 0 with Faraday rotation and 1.000000 with optical activity——one-way passage needs a non-reciprocal medium.A 1 degree rotation error leaks 3.045865×10^{-4} backward, an isolation of 35.162894 dB, and at 1 T with a Verdet constant of 134 rad/(T m), 45 degrees takes 5.861180 mm.The separator is whether the sense of rotation is set by the material's handedness relative to the direction of travel or by the direction of an applied magnetic field——"an element that turns light by 45 degrees" does not say whether it can stop light until one says which.Placed among the earlier papers——Paper 335 showed a round trip changing an exponent; this counts a round trip cancelling or adding a rotation.To be honest——losses, wavelength dependence, the electro-optic effect and real devices are not treated.**

References

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