Stable Spin Axes Number Two When Energy Is Kept and One When It Leaks

— with energy conserved only a disturbance about the intermediate axis grows, at **7.658331** per second, but leaking energy at fixed angular momentum settles even a start about the smallest axis onto the largest — the separator is whether energy leaks while the size of the angular momentum is kept — [Paper 778]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— From thinking to syncing. —

Abstract

Asking whether something spinning is stable, there are three axes. For a rigid body with no forces, rotation about the axes of smallest and largest moment of inertia is stable and only the intermediate axis fails: this is known.What happens to the number of stable axes when a mechanism is added that lets only energy leak while keeping the size of the angular momentum is counted.No new theorem or law is claimed. First, when energy is kept, two axes are stable ——the largest deviation over 30 s is 0.000163 about the smallest axis and 0.000142 about the largest (Section 2). **Second, only the intermediate axis fails——the deviation spreads to 1.259564 and the main component flips 11 times** (Section 2). **Third, the failure grows at the rate of the formula——formula 7.658331, numerical 7.658647 per second** (Section 2). **Fourth, and this is the core. When energy leaks, the smallest axis is no longer stable——the same start drops below the boundary 1.246455 at 23.275 s and cannot return** (Section 3). **Fifth, the destination is the largest axis——the final share of the largest-axis component is 1.000000000** (Section 3). **Sixth, the separator is whether energy leaks while the size of the angular momentum is kept——"rotation about the long side is stable" has no truth value until one says whether energy leaks** (Section 4). **Keywords**: intermediate axis theorem, rigid-body rotation, Euler's equations, moment of inertia, energy dissipation, rotational stability.

Scope of this paper (scope note): **No new theorem or law is claimed——Euler's equations, the instability of the intermediate axis alone (known as the tennis racket theorem or the Dzhanibekov effect), and the drift to the axis of largest moment of inertia when energy leaks (known from the attitude change of the satellite Explorer 1) are all known (Landau and Lifshitz 1976, Goldstein et al. 2002).The form of dissipation is a model——the simplest term that keeps the size of the angular momentum and lowers only the energy was used. What drains energy inside a real body (flexing, sloshing liquid, friction) is not treated.No external torque——neither gravity nor air resistance is included.Values are representative——a uniform plate of mass 0.2 kg measuring $0.15 \times 0.075 \times 0.008$ m.No claim about real satellites——which craft turned how and when is not treated.Relation to earlier papers**: **Paper 586 showed that whether a top stands or falls is decided not by how fast it spins but by one threshold——its scope note excluded friction and stated that spin-down was not treated. There a top under gravity was counted without friction. Here dissipation is added to a force-free rigid body, and the stable axes are counted as dropping from two to one. Above or below a threshold, and the number of stable axes.**What is added is **checking the deviation, flip count, oscillation period and growth rate of the three axes in two ways, by integration and by formula, adding dissipation that keeps the size of the angular momentum and deriving the time 23.275 s at which a smallest-axis start drops below the boundary, showing that the destination is the largest axis (1.000000000), and placing the separator on whether energy leaks.**

1. Notation and the Question

Setting: **let the principal moments be $I_1 < I_2 < I_3$, and write the angular momentum $\mathbf{L}$ and angular velocity $\omega_i = L_i/I_i$. The second term on the right is the dissipation: it keeps $|\mathbf{L}|$ and lowers only the energy E.**

$$\dot{\mathbf{L}} = \mathbf{L} \times \vec{\omega} + \frac{\gamma}{|\mathbf{L}|}\,\mathbf{L} \times \left(\mathbf{L} \times \vec{\omega}\right), \qquad \frac{d}{dt}|\mathbf{L}|^2 = 0, \qquad \dot{E} = -\frac{\gamma}{|\mathbf{L}|}\left(|\mathbf{L}|^2|\vec{\omega}|^2 - (\mathbf{L}\cdot\vec{\omega})^2\right) \le 0$$

With $\gamma = 0$ these are Euler's equations. The energy lies between $|\mathbf{L}|^2/(2I_3)$ (lowest) and $|\mathbf{L}|^2/(2I_1)$ (highest), and $|\mathbf{L}|^2/(2I_2)$ is the boundary.

Question: **spinning a plate about its three axes, how many axes are stable? Does that change when energy leaks?**

2. When Energy Is Kept

Computation 2.1: **a plate of mass 0.2 kg measuring $0.15 \times 0.075 \times 0.008$ m ($I_1 = 9.481667 \times 10^{-5}$, $I_2 = 3.760667 \times 10^{-4}$, $I_3 = 4.687500 \times 10^{-4}$ kg m^2) was spun at 10 rad/s about each axis with a 0.001 rad/s disturbance in the other two components and integrated for 30 s. The deviation is the size of the other angular-velocity components divided by 10 rad/s.**

axis	largest deviation	flips of main component	oscillation period (formula, numerical)
smallest (about the long side)	0.000163	0	0.813466 s, 0.813466 s
intermediate	1.259564	11	—
largest (normal to the face)	0.000142	0	0.637319 s, 0.637319 s

Observation 2.2

when energy is kept, two axes are stable.The deviations about the smallest and largest axes stay at 0.000163 and 0.000142, the same order as the disturbance put in, and the oscillation periods agree with the formula to a relative difference of 1.617×10^{-9}——the deviation does not grow; it only oscillates in place.

Observation 2.3

only the intermediate axis fails.The deviation spreads to 1.259564; the main component first flips at 1.550475 s and flips 11 times, every 2.825624 s on average. The growth rate of the deviation is given below as 7.658331 per second, and the numerical slope is 7.658647 per second (relative difference 4.134×10^{-5}).

$$\lambda = \omega \sqrt{\frac{(I_3 - I_2)(I_2 - I_1)}{I_1 I_3}}$$

Only for the middle axis is the quantity under the root positive.

3. When Energy Leaks

Computation 3.1: **with $\gamma = 0.05$, the same start about the smallest axis was integrated for 150 s. The initial energy is 4.943750 times the lowest ($I_3/I_1 = 4.943751$), and the boundary is 1.246455 times the lowest.**

time	share of smallest-axis component	share of largest-axis component	energy / lowest
25 s	0.144174	0.688450	1.206497
50 s	0.000996	0.999997	1.000005
75 s	0.000006	1.000000	1.000000
150 s	0.000000	1.000000	1.000000

Observation 3.2

this is the core: when energy leaks, the smallest axis is no longer stable.The same start that stayed at a deviation of 0.000163 without dissipation loses energy and drops below the boundary 1.246455 at 23.275 s ——energy never rises, so once below the boundary it cannot return to the smallest axis.

Observation 3.3

the destination is the largest axis.At 50 s the share of the largest-axis component is 0.999997, finally 1.000000000. The size of the angular momentum drifts by 1.993×10^{-11}, and the largest energy rise in a step is 2.168×10^{-19} J, the size of rounding——for a given angular momentum, rotation about the largest axis has the lowest energy.

4. The Separator

Observation 4.1

the separator is whether energy leaks while the size of the angular momentum is kept.If it does not, two axes, smallest and largest, are stable; if it does, only the largest remains——“rotation about the long side is stable” has no truth value until one says whether energy leaks.

Observation 4.2

beside Paper 586.Paper 586 showed without friction that a top under gravity stands only above one threshold——here dissipation in the role of friction is added to a force-free rigid body, and the membership of the stable set itself is counted as changing.

Observation 4.3

what is not said.The dissipation is a model, and neither what drains energy in a real body nor the attitude of real satellites is treated——what is shown is that the number of stable axes splits into two and one by whether energy leaks.

5. Roll Call and Closing

1. **smallest-axis deviation 0.000163** (Computation 2.1): **stable.**
2. **largest-axis deviation 0.000142** (Computation 2.1): **stable.**
3. **intermediate-axis deviation 1.259564** (Observation 2.3): **fails.**
4. **11 flips, every 2.825624 s** (Observation 2.3): **turning over again and again.**
5. **growth rate 7.658331 and 7.658647** (Observation 2.3): **as the formula says.**
6. **oscillation periods 0.813466 and 0.637319 s** (Computation 2.1): **matching the formula.**
7. **below the boundary 1.246455 at 23.275 s** (Observation 3.2): **the core.**
8. **largest-axis share 1.000000000** (Observation 3.3): **the destination.**
9. **initial energy 4.943750 times the lowest** (Computation 3.1): **a smallest-axis start.**
10. **whether energy leaks** (Observation 4.1): **the separator; Paper 586.**

Closing: **Stable spin axes number two when energy is kept and one when it leaks.Spinning a force-free plate at 10 rad/s, the largest deviation over 30 s stays at 0.000163 about the smallest axis and 0.000142 about the largest, while only the intermediate axis fails, reaching 1.259564 and flipping 11 times——the failure grows at 7.658331 by formula and 7.658647 numerically, and the oscillation periods also match the formula.Letting only energy leak while keeping the size of the angular momentum, the same smallest-axis start drops below the boundary 1.246455 at 23.275 s, cannot return, and settles with a largest-axis share of 1.000000000——for a given angular momentum, the largest axis has the lowest energy.The separator is whether energy leaks while the size of the angular momentum is kept.Placed among the earlier papers——Paper 586 counted a top under gravity without friction; this adds**

dissipation to a force-free rigid body and counts the number of stable axes changing.To be honest——the dissipation is a model, and neither the mechanism inside real bodies nor satellite attitude is treated.

References

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