Where the Tides End Up Is Set by Angular Momentum Alone, and How Long It Takes Is Set by Dissipation

— Keeping the total angular momentum of Earth and Moon, 3.442739×10^{34} kg m^2/s, and matching Earth's day to the Moon's orbit gives an end state at 1.440675 times the present distance with a period of 47.180649 days — the separator is whether one asks for the destination or for the time of arrival — [Paper 783]

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— From thinking to syncing. —

Abstract

Tides slow Earth's rotation and push the Moon away. Where this change stops and when it gets there look like one question.The destination is derived from conserved angular momentum and the time from the present recession rate, and the two questions are counted as being set by different quantities.No new theorem or law is claimed. First, the destination is where Earth's day and the Moon's orbit coincide——a distance 1.440675 times the present one and a period of 47.180649 days (Section 2). **Second, that destination is fixed without using the strength of dissipation——it is solved from the total angular momentum 3.442739×10^{34} kg m^2/s alone, and solving for the period instead differs by 9.948×10^{-14} days** (Section 2). **Third, the time to get there is set by dissipation——tracing the present recession with constant dissipation gives 1.506629×10^{10} years** (Section 3). **Fourth, and this is the core. Tracing backward with the same dissipation, time runs short——the Moon would have been at three Earth radii 1.548123×10^{9} years ago, only 0.343265 of its age** (Section 3). **Fifth, the close distances are passed almost instantly——from ten to three Earth radii takes 13058.608413 years, 8.435×10^{-6} of the whole** (Section 3). **Sixth, the separator is whether one asks for the destination or for the time of arrival——"tides will eventually stop the Moon" does not settle what is known until one says which is being asked** (Section 4). **Keywords**: tides, angular momentum, lunar recession, synchronous rotation, tidal dissipation, Earth's rotation.

Scope of this paper (scope note): **No new theorem or law is claimed——tides slowing Earth's rotation and pushing the Moon away, the existence of an end state where Earth's day and the Moon's orbit coincide under conserved angular momentum, and the backward extrapolation of the present recession with constant dissipation coming out shorter than the Moon's age are all known (MacDonald 1964, Murray and Dermott 1999).Dissipation is held constant——only the simplest model, with recession speed proportional to distance to the $-11/2$, is used; changes of dissipation through ocean shape or resonance are not treated.Solar tides are not included——only the angular momentum of Earth and Moon is conserved.Values are representative——the masses, distance, Earth's moment-of-inertia factor and sidereal day are aligned with Paper 686, with a recession speed of 3.82 cm per year, a lunar age of 4.51×10^9 years and a lunar moment-of-inertia factor of 0.394.The Moon's origin is not treated——as in Paper 686, which hypothesis is right is not stated.Relation to earlier papers**: **Paper 686 showed that what narrows the origin of the Moon is not composition but one quantity that cannot be changed, angular momentum——its scope note stated that time evolution, including the Moon's tidal recession, was not followed, comparing only the present partition with the end where everything is returned to Earth's spin. Here the same conserved quantity gives the opposite end, where Earth's day and the Moon's orbit coincide, and the time to it is counted as set by dissipation.Paper 406 showed that the same face being seen is the evidence that the Moon rotates——its scope note stated that the tidal mechanism behind synchronous rotation was not entered. Here the destination of that mechanism is counted on Earth's side.**What is added is **solving the end state with two unknowns, distance and period, and matching them**, **confirming that the end state has lower energy than the present**, **deriving the times two ways, by closed formula and numerical integration, and showing that the backward trace gives only 0.343265 of the Moon's age**, and **placing the separator on destination versus time of arrival**.

1. Notation and the Question

Setting: **let I_E and I_M be the moments of inertia of Earth and Moon, μ the reduced mass, M the total mass, a the distance and $n(a) = \sqrt{GM/a^3}$ the Moon's mean motion. The total angular momentum L is conserved, and at the end state Earth's spin rate matches n.**

$$L = I_E\,\omega_E + I_M\,n + \mu\sqrt{GMa}, \qquad (I_E + I_M)\,n(a_f) + \mu\sqrt{GMa_f} = L$$

For time, the recession speed is taken proportional to distance to the $-11/2$ (fixed by the present speed v_0).

$$\frac{da}{dt} = v_0\left(\frac{a_0}{a}\right)^{11/2}, \qquad t(a_1 \to a_2) = \frac{a_2^{13/2} - a_1^{13/2}}{\frac{13}{2}\,v_0\,a_0^{11/2}}$$

Question: **where does the tidal change stop, and when does it get there?**

2. The Destination

Computation 2.1: **the present angular momentum was derived and the end state solved by bisection with distance as the unknown. The same end state was also solved with period as the unknown.**

quantity	value
Earth's spin angular momentum	5.847478×10^{33} kg m^2/s
Moon's spin angular momentum	2.328617×10^{29} kg m^2/s
orbital angular momentum	2.857968×10^{34} kg m^2/s
total	3.442739×10^{34} kg m^2/s
end-state distance	5.537941×10^{8} m (1.440675 times present)
end-state period	47.180649 days (present sidereal month 27.284423 days)
difference when solved for period	9.948×10^{-14} days
the other, inner synchronous position	2.294357 Earth radii, 4.855742 hours

> **Observation 2.2**
>
> **the destination is where Earth's day and the Moon's orbit coincide.The distance becomes 1.440675 times the present one, and both the day and the month become 47.180649 days——this point is solved from the total angular momentum alone; the strength of dissipation appears nowhere.**

> **Observation 2.3**
>
> **the end state has lower energy than the present.The mechanical energy is 1.751155×10^{29} J now and -2.634167×10^{28} J at the end, 2.014572×10^{29} J lower——it is consistent as the place dissipation drives toward while shedding energy at fixed angular momentum.**

3. The Time to Get There

Computation 3.1: **the present recession speed of 0.0382 m per year was traced with the constant-dissipation model, and the times to the future end state and back to close past distances derived by closed formula and by numerical integration.**

interval	closed formula	numerical integration
now → end state	1.506629×10^{10} years	1.506629×10^{10} years
three Earth radii → now	1.548123×10^{9} years	1.548123×10^{9} years
ten Earth radii → now	1.548110×10^{9} years	—

Observation 3.2

the time to get there is set by dissipation.With the present dissipation the end state takes 1.506629×10^{10} years——this time is inversely proportional to the recession speed; halve the dissipation and it doubles. The destination stays; only the arrival time moves.

Observation 3.3

this is the core: tracing backward with the same dissipation, time runs short.The Moon would have been at three Earth radii 1.548123×10^{9} years ago, only 0.343265 of the Moon's age of 4.51×10^{9} years——to match the age, the average past dissipation must be 0.343265 times the present. The destination calculation is untouched by this discrepancy.

Observation 3.4

the close distances are passed almost instantly.From ten to three Earth radii takes 13058.608413 years, 8.435×10^{-6} of the whole——because distance enters to the $13/2$ power, nearly all the time is spent on the far side.

4. The Separator

Observation 4.1

the separator is whether one asks for the destination or for the time of arrival.The destination (1.440675 times, 47.180649 days) is set by angular momentum alone; the arrival time (1.506629×10^{10} years) is set by dissipation, and that dissipation, traced backward, does not match the Moon's age——"tides will eventually stop the Moon" does not settle what is known until one says which is being asked.

Observation 4.2

beside Paper 686 and Paper 406.Paper 686 compared the end where the same angular momentum is returned entirely to Earth's spin, and Paper 406 showed how synchronous rotation appears——here the same conserved quantity gives the opposite end, and the time to it is counted as set by a different quantity.

Observation 4.3

what is not said.Neither solar tides, nor how dissipation changed in the past, nor the Moon's origin is treated ——what is shown is that the destination and the time of arrival are set by different quantities.

5. Roll Call and Closing

1. **total angular momentum 3.442739×10^{34}** (Computation 2.1): **the conserved quantity.**
2. **end-state distance 1.440675 times** (Observation 2.2): **the destination.**
3. **end-state period 47.180649 days** (Observation 2.2): **day and month coincide.**
4. **difference between two solutions 9.948×10^{-14} days** (Computation 2.1): **no dissipation used.**
5. **energy difference 2.014572×10^{29} J** (Observation 2.3): **the end state is lower.**
6. **1.506629×10^{10} years to the end** (Observation 3.2): **set by dissipation.**
7. **1.548123×10^{9} years back** (Observation 3.3): **time runs short.**
8. **0.343265 of the Moon's age** (Observation 3.3): **the core.**
9. **13058.608413 years from ten to three radii** (Observation 3.4): **passed in an instant.**
10. **destination or arrival time** (Observation 4.1): **the separator; Paper 686 and Paper 406.**

Closing: **Where the tides end up is set by angular momentum alone, and how long it takes is set by dissipation.Keeping the total angular momentum of Earth and Moon, 3.442739×10^{34} kg m^2/s, and matching Earth's day to the Moon's orbit gives an end state at 1.440675 times the present distance with a period of 47.180649 days, reached also when solving for the period——this end state does not move even without knowing the strength of dissipation.Tracing the present recession of 3.82 cm per year with constant dissipation takes 1.506629×10^{10} years to reach it, while tracing backward puts the Moon at three Earth radii 1.548123×10^{9} years ago, only 0.343265 of the Moon's age of 4.51×10^{9} years——the arrival time moves with dissipation, and the average past dissipation must have been smaller than today's.The separator is whether one asks for the destination or for the time of arrival.Placed among the earlier papers——Paper 686 compared the end where angular momentum is returned, and Paper 406 showed how synchronous rotation appears; this counts the opposite end of the same conserved quantity and the time to reach it.To be honest——neither solar tides, nor past changes in dissipation, nor the Moon's origin are treated.**

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