The Roche Limit Is Not One Distance: It Differs by a Factor of Two Depending on What Holds the Satellite Together

— in units of the planet's radius times the cube root of the density ratio, the limit is **1.259921** for a solid resisting tides alone and **2.455** for a fluid, a factor of **1.948535** apart — the separator is whether a satellite holds its shape by fluid self-gravity, solid self-gravity or material strength — [Paper 784]

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— *From thinking to syncing.* —

Abstract

A satellite that comes too close to its planet is torn apart by tides. That distance is called the Roche limit and is spoken of as one value.How many distances exist under the same name, by differences in what holds a satellite together, is counted.No new theorem or law is claimed. First, even for solids there are two limits——1.259921 for a solid resisting tides alone and 1.442250 for a synchronously rotating solid (in units of planet radius times the cube root of the density ratio) (Section 2). **Second, and this is the core. For a fluid, the limit is about twice as far——the fluid's 2.455 is 1.948535 times that of the tides-only solid** (Section 2). **Third, Saturn's rings sit at the edge of the fluid limit——the fluid limit for ice (density 917) is 2.229704 Saturn radii, the outer edge of the A ring 2.269446, and for porous particles (500) the limit is 2.729274** (Section 3). **Fourth, this limit does not apply to small moons ——the self-gravitational pressure of Pan (2.216500), inside the rings, is 4446.543499 Pa, 4.446543×10^{-3} of the strength of ice** (Section 3). **Fifth, the closed formula is an approximation for small satellites——a relative difference of 5.273×10^{-5} for a satellite of 1 km radius, but 1.633126 times for one as large as the Moon (formula 1.488482)** (Section 2). **Sixth, the separator is whether a satellite holds its shape by fluid self-gravity, solid self-gravity or material strength——"inside the Roche limit a satellite breaks up" has no truth value until one says what holds the satellite together** (Section 4). **Keywords**: Roche limit, tidal force, self-gravity, Saturn's rings, material strength, satellites.

Scope of this paper (scope note): No new theorem or law is claimed——setting the breakup distance by balancing tides against self-gravity (Roche), the coefficient 2.455 for fluid satellites (Chandrasekhar), the solid coefficients, and small bodies holding their shape by material strength are all known (Chandrasekhar 1969, Murray and Dermott 1999).The fluid coefficient is not derived——2.455 is used as input, and equilibrium figures are not computed.The origin of the rings is not treated——how the rings formed or why they lie where they do is not stated.Values are representative——densities of Earth 5514, Moon 3344, Saturn 687, ice 917 and porous particles 500 kg/m^3, ice strength 10^6 Pa, and Pan's radius 14.1 km and density 400 kg/m^3 are for comparison; measurements are not claimed.Strength is compared by central pressure only——fracture modes and internal structure are not treated.Relation to earlier papers: Paper 505 showed that the "sphere of influence" is three spheres, and dividing the mass ratio by 1000 shrinks them differently——its scope note stated that nothing beyond the three was counted and that the Roche limit and gravitational capture radius were not included. There the sphere of influence split into three by what the gravity is balanced against. Here the same "Roche limit" is counted as splitting into three by what holds the satellite together.What is added is setting the three coefficients side by side and showing fluid and solid differ by **1.948535**, solving the exact balance by bisection and confirming it returns to the closed formula for small satellites and departs for large ones, placing Saturn's rings and small moons against the three limits, and placing the separator on what holds the shape.

1. Notation and the Question

Setting: let the planet have radius R_p and density ρ_p, and the satellite density ρ_s. The breakup distance d takes a form in which only the coefficient c differs by how the satellite holds together.

$$d = c\,R_p\left(\frac{\rho_p}{\rho_s}\right)^{1/3}, \qquad c = \begin{cases} 2^{1/3} & \text{solid, tides only} \\ 3^{1/3} & \text{solid, synchronous rotation} \\ 2.455 & \text{fluid} \end{cases}$$

The solid coefficients come from the condition that tides (and centrifugal force) at the satellite's surface match its self-gravity, an approximation for small satellites. The exact balance solves the following (m_s is the satellite mass and R_s its radius).

$$GM_p\left[\frac{1}{(d-R_s)^2}-\frac{1}{d^2}\right]=\frac{Gm_s}{R_s^2}$$

Question: is the distance at which tides break a satellite fixed as one value?

2. Three Coefficients

Computation 2.1: with the three coefficients, limits were derived for Earth and Moon, Saturn and ice, and Saturn and porous ice particles.

pair	solid, tides only	solid, synchronous	fluid
Earth and Moon (3344)	1.488482	1.703886	2.900358
Saturn and ice (917)	1.144298	1.309894	2.229704
Saturn and porous particles (500)	1.400680	1.603378	2.729274

Units are planet radii. Fluid over tides-only solid = 1.948535; fluid over synchronous solid = 1.702202.

Computation 2.2: the exact balance was solved by bisection and compared with the closed formula.

pair and mode	closed formula	satellite radius 1 km	satellite radius 1737 km
Earth and Moon, tides only	1.488482	1.488560	1.633126
Earth and Moon, synchronous	1.703886	1.703939	1.804870
Saturn and ice, tides only	1.144298	1.144306	1.158833
Saturn and ice, synchronous	1.309894	1.309900	1.319645

> **Observation 2.3**
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> **even for solids there are two limits.Depending on whether only tides are counted or the centrifugal force of synchronous rotation is added, the coefficient splits into 1.259921 and 1.442250——the same “solid limit” moves by the ratio of the coefficients depending on what is counted as a force.**

> **Observation 2.4**
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> **this is the core: for a fluid, the limit is about twice as far.The fluid coefficient 2.455 is 1.948535 times that of the tides-only solid——a fluid, stretched by tides as it changes shape and unable to resist by rigidity, breaks up much farther out.**

Observation 2.5

the closed formula is an approximation for small satellites.For a satellite of 1 km radius it agrees with the exact balance to a relative difference of 5.273×10^{-5}, but for one the size of the Moon the exact value is 1.633126, departing from the formula's 1.488482.

3. Saturn's Rings and a Small Moon

Computation 3.1: **in units of Saturn's radius, the positions of rings and moons were set against the three limits, and a small moon's self-gravitational central pressure compared with the strength of ice.**

quantity	value
outer edge of the A ring	**2.269446** Saturn radii
Pan (small moon within the rings)	**2.216500** Saturn radii
Mimas	**3.078566** Saturn radii
Pan's self-gravitational central pressure	**4446.543499** Pa
strength of ice (representative)	$\mathbf{10^6}$ Pa
radius where self-gravity pressure matches strength	**92.235583** km

Observation 3.2

Saturn's rings sit at the edge of the fluid limit.The A ring's outer edge at 2.269446 lies just outside the fluid limit 2.229704 for ice of 917, and inside the limit 2.729274 for porous particles of 500. Mimas at 3.078566 is outside every limit——whether the ring edge is inside or outside the limit swaps with how tightly the particles are packed.

Observation 3.3

this limit does not apply to small moons.Pan lies at 2.216500, inside the fluid limit for ice, yet its self-gravitational central pressure is 4446.543499 Pa, only 4.446543×10^{-3} of the strength of ice. Self-gravity pressure matches strength only from a radius of 92.235583 km——smaller moons hold their shape not by self-gravity but by material strength.

4. The Separator

Observation 4.1

the separator is whether a satellite holds its shape by fluid self-gravity, solid self-gravity or material strength.A fluid gives 2.455, a solid 1.259921 or 1.442250, and for small moons held by strength the limit itself does not bite——"inside the Roche limit a satellite breaks up" has no truth value until one says what holds the satellite together.

Observation 4.2

beside Paper 505.Paper 505 showed that the same "sphere of influence" splits into three by what a planet's gravity is balanced against, leaving the Roche limit out——here the same "Roche limit" is counted as splitting into three by how the satellite holds together.

Observation 4.3

what is not said.Neither the derivation of the fluid coefficient, nor the origin of the rings, nor measured densities or strengths of real moons are treated——what is shown is that under one name there are as many distances as there are ways of holding together.

5. Roll Call and Closing

1. **tides-only solid 1.259921** (Computation 2.1): **the first limit.**
2. **synchronous solid 1.442250** (Observation 2.3): **the second limit.**
3. **fluid 2.455, ratio 1.948535** (Observation 2.4): **the core.**

4. **1.633126 for a Moon-sized satellite** (Observation 2.5): **the formula is a small-satellite approximation.**
5. **fluid limit for ice 2.229704** (Computation 2.1): **on Saturn's side.**
6. **A ring outer edge 2.269446** (Observation 3.2): **at the edge of the limit.**
7. **limit for porous particles 2.729274** (Observation 3.2): **swaps with packing.**
8. **Pan's pressure 4446.543499 Pa** (Observation 3.3): **4.446543×10^{-3} of strength.**
9. **matching radius 92.235583 km** (Observation 3.3): **the border of the world of strength.**
10. **what holds the shape** (Observation 4.1): **the separator; Paper 505.**

Closing: **The Roche limit is not one distance: it differs by a factor of two depending on what holds the satellite together.Taking planet radius times the cube root of the density ratio as the unit, the limit is 1.259921 for a tides-only solid, 1.442250 for a synchronous solid and 2.455 for a fluid, so fluid and solid differ by 1.948535——the closed formula approximates small satellites; for one as large as the Moon the exact balance gives 1.633126.Saturn's fluid limit for ice is 2.229704 Saturn radii, and the A ring's outer edge at 2.269446 lies just outside it, inside for porous particles. Pan, within the rings at 2.216500, has a self-gravitational central pressure of 4446.543499 Pa, only 4.446543×10^{-3} of the strength of ice, and moons smaller than 92.235583 km hold their shape by strength——the limit bites only on satellites held together by self-gravity.The separator is whether a satellite holds its shape by fluid self-gravity, solid self-gravity or material strength.Placed among the earlier papers——Paper 505 counted the sphere of influence splitting into three by what it is balanced against; this counts the Roche limit splitting into three by how the satellite holds together.To be honest——the fluid coefficient is an input, and neither the rings' origin nor the strength of real moons is treated.**

References

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