Ice-Core Ages Are Set by Snowfall Near the Top and by the Flow Model Near the Bottom

— at **2900** m they are **3.401197×10^{5}** and **1.870904×10^{6}** years, ratio **5.500720**, with **1000** and **19200** years packed into one metre of ice — the separator is from what depth the layers begin to thin (the shape of the ice flow) — [Paper 786]

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September 2026

— From thinking to syncing. —

Abstract

An ice sheet is built from snow laid down each year, so depth corresponds to age. But ice flows sideways under its own weight, and deeper layers are stretched thinner.With two models that thin the layers differently, ages at each depth are compared, and how far down the age is set by accumulation and from where it is set by the flow model is counted.No new theorem or law is claimed. First, at shallow depths the two models give nearly the same age——1.823216×10^{4} and 1.868919×10^{4} years at 500 m, ratio 1.025067 (Section 2). **Second, the two first differ by ten percent above half the thickness——at 1309 m depth, 0.436333 of the thickness** (Section 2). **Third, and this is the core. Near the bottom, the age changes several times over with the model——3.401197×10^{5} and 1.870904×10^{6} years at 2900 m, ratio 5.500720** (Section 2). **Fourth, how tightly the scale is packed also differs by model——one metre of ice at 2900 m holds 1000 or 19200 years** (Section 2). **Fifth, closer still to the bed, the gap keeps widening ——ratio 33.575796 at 2990 m** (Section 2). **Sixth, the separator is from what depth the layers begin to thin (the shape of the ice flow)——"the ice at this depth is so many years old" is set by accumulation near the top, but near the bottom it is not settled until one names the flow model** (Section 3). **Keywords**: ice cores, age scale, Nye model, Dansgaard-Johnsen model, ice flow, annual layers.

Scope of this paper (scope note): **No new theorem or law is claimed——the Nye model with layers thinning in proportion to depth, the Dansgaard--Johnsen model with rapid thinning near the bed, their age formulas, and the growing dependence of dating on flow assumptions toward the bed are all known (Dansgaard and Johnsen 1969, Cuffey and Paterson 2010).The models are steady——accumulation and ice thickness are constant, and basal melting, firn densification and climate-driven changes in accumulation are left out.Values are representative——thickness 3000 m, accumulation (ice) 0.03 m per year and kink height 1200 m are for comparison; ages of real cores are not claimed.Other dating clues are not treated——matching with volcanic ash, isotopes or orbital periods is not treated.Which model is right is not said——only how much the difference between the two models affects age at each depth is examined.Relation to earlier papers**: **Paper 432 showed that the strongest period in the record is the weakest in the forcing——its scope note stated that frequency analysis methods were not discussed, and that dating errors and non-stationarity of the record were not its subject. There the periods appearing in the glacial record were compared. Here the age scale of that record itself is counted as set by accumulation near the top and by the flow model near the bottom.**What is added is **tabulating the two models' ages at ten depths and showing the ratio widening from 1.025067 to 33.575796**, **deriving the first depth of ten-percent divergence, 1309 m**, **deriving ages two ways, by closed formula and numerical integration**, and **placing the separator on from what depth the layers thin**.

1. Notation and the Question

Setting: **let H be the ice thickness, b the accumulation rate, z the height above the bed and $d = H - z$ the depth below the surface. Integrating the reciprocal of the layer thickness (the rate at which ice moves down) $\lambda(z)$ gives the age.**

$$t(d) = \int_{H-d}^{H} \frac{dz}{\lambda(z)}, \qquad \lambda_{\mathrm{N}}(z) = b\,\frac{z}{H}, \qquad t_{\mathrm{N}}(d) = \frac{H}{b}\ln\frac{H}{H-d}$$

The Dansgaard--Johnsen model thins linearly above a kink height h and as the square of height below it.

$$\lambda_{\mathrm{DJ}}(z) = b\,\frac{z - h/2}{H - h/2} \quad (z \ge h), \qquad \lambda_{\mathrm{DJ}}(z) = b\,\frac{z^2}{h(2H - h)} \quad (z < h)$$

Take $H = 3000$ m, $b = 0.03$ m per year and $h = 1200$ m.

Question: **is age fixed uniquely by depth in an ice core?**

2. Ages in the Two Models

Computation 2.1: **at ten depths, the ages in the two models, their ratio, and the years in one metre of ice were derived. Ages were derived two ways, by closed formula and by numerical integration in a logarithmic variable, with a largest relative difference of 2.176×10^{-14}.**

depth	Nye age	DJ age	ratio	years per metre (Nye, DJ)
500 m	1.823216×10^{4} yr	1.868919×10^{4} yr	1.025067	40.000000, 42.105263
1000 m	4.054651×10^{4} yr	4.311972×10^{4} yr	1.063463	50.000000, 57.142857
1500 m	6.931472×10^{4} yr	7.846634×10^{4} yr	1.132030	66.666667, 88.888889
1800 m	9.162907×10^{4} yr	1.109035×10^{5} yr	1.210353	83.333333, 133.333333
2000 m	1.098612×10^{5} yr	1.429035×10^{5} yr	1.300764	100.000000, 192.000000
2500 m	1.791759×10^{5} yr	3.349035×10^{5} yr	1.869132	200.000000, 768.000000
2800 m	2.708050×10^{5} yr	9.109035×10^{5} yr	3.363688	500.000000, 4800.000000
2900 m	3.401197×10^{5} yr	1.870904×10^{6} yr	5.500720	1000.000000, 19200.000000
2950 m	4.094345×10^{5} yr	3.790904×10^{6} yr	9.258878	2000.000000, 76800.000000
2990 m	5.703782×10^{5} yr	1.915090×10^{7} yr	33.575796	10000.000000, 1920000.000000

Observation 2.2

at shallow depths the two models give nearly the same age.The ratio at 500 m is 1.025067——near the surface both models have layer thickness close to the accumulation rate, so age is set almost entirely by accumulation.

Observation 2.3

the two first differ by ten percent above half the thickness.The first depth where the ratio exceeds 1.1 is 1309 m (0.436333 of the thickness), shallower than the kink (at 1800 m depth)——even above the kink, the difference in slope of the two straight lines accumulates into age.

Observation 2.4

this is the core: near the bottom, the age changes several times over with the model.At 2900 m the ages are 3.401197×10^5 and 1.870904×10^6 years, ratio 5.500720, and at 2990 m the ratio is 33.575796——the Nye age grows only logarithmically with depth, while in Dansgaard--Johnsen it grows inversely with height above the bed, so the gap keeps widening.

Observation 2.5

how tightly the scale is packed also differs by model.At 2900 m, one metre of ice holds 1000 or 19200 years ——how much time the same one-metre sample averages also changes with the flow assumption, by the ratio of these two values.

3. The Separator

Observation 3.1

the separator is from what depth the layers begin to thin (the shape of the ice flow).Near the surface every model has layer thickness close to accumulation, so age is fixed once; near the bed, how the layers thin changes age by 5.500720 and 33.575796 times——"the ice at this depth is so many years old" is set by accumulation near the top, but near the bottom it is not settled until one names the flow model.

Observation 3.2

beside Paper 432.Paper 432 showed that the strongest period in the glacial record is the weakest in the forcing, leaving dating error aside——here the age scale of that record is counted as set by different quantities at different depths.

Observation 3.3

what is not said.Neither ages of real cores, nor which model is right, nor other dating clues are treated—— what is shown is that the age scale is set by different things at the top and near the bottom.

4. Roll Call and Closing

1. **ratio 1.025067 at 500 m** (Computation 2.1): **one age near the top.**
2. **ten-percent depth 1309 m** (Observation 2.3): **0.436333 of the thickness.**
3. **ratio 1.300764 at 2000 m** (Computation 2.1): **beginning to diverge.**
4. **ages 3.401197×10^{5} and 1.870904×10^{6} years at 2900 m** (Computation 2.1): **two scales.**
5. **ratio 5.500720** (Observation 2.4): **the core.**
6. **ratio 33.575796 at 2990 m** (Observation 2.4): **widening further.**
7. **1000 and 19200 years per metre** (Observation 2.5): **packing differs.**
8. **relative difference 2.176×10^{-14} from integration** (Computation 2.1): **two ways agree.**
9. **Nye age 6.931472×10^{4} years at 1500 m** (Computation 2.1): **a scale of $\ln 2$.**
10. **from what depth the layers thin** (Observation 3.1): **the separator; Paper 432.**

Closing: **Ice-core ages are set by snowfall near the top and by the flow model near the bottom.In ice 3000 m thick with accumulation of 0.03 m per year, the age at 500 m is 1.823216×10^{4} years in the Nye model and 1.868919×10^{4} years in the Dansgaard--Johnsen model, nearly the same at a ratio of 1.025067——the two first differ by ten percent at 1309 m.At 2900 m they are 3.401197×10^{5} and 1.870904×10^{6} years, ratio 5.500720, with 1000 and 19200 years per metre of ice, and at 2990 m the ratio widens to 33.575796——age near the bed is set not by accumulation but by the assumption of how the layers thin.The separator is from what depth the layers begin to thin (the shape of the ice flow).Placed among the earlier papers——Paper 432 compared the periods appearing in the record; this counts the age scale of that record as set by different quantities at different depths.To be honest——the models are steady, and neither the ages of real cores nor which model is right is treated.**

References

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