Incoming and Outgoing Light Part at 4.245712 μm, and This Dividing Line Does Not Depend on the Choice of Variable

— The normalized light distributions of the Sun at **5772** K and Earth at **288** K cross at a wavelength of **4.245712** μm, and their overlap is only $\mathbf{1.081347 \times 10^{-2}}$ — the separator is whether a statement compares distributions (shares, crossings) or locates the peak of one distribution — [Paper 788]

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— *From thinking to syncing.* —

Abstract

Earth receives sunlight and sends its own light to space. The outgoing light is sometimes said to peak in the middle of the 8 to 13 μm "window" where the atmosphere is transparent.Treating Sun and Earth as black bodies, the dividing line and overlap of the two lights, the share entering the window, and the peak positions are counted, separated into what depends on the choice of variable and what does not.No new theorem or law is claimed. First, and this is the core. Incoming and outgoing light part at a single wavelength——the two normalized distributions cross at 4.245712 μm, with an overlap of $\mathbf{1.081347 \times 10^{-2}}$ (Section 3). **Second, this dividing line and overlap do not depend on the choice of variable——both distributions undergo the same change of variable, so neither the crossing point nor the integral of the smaller one changes; two derivations differ by $\mathbf{5.122 \times 10^{-9}}$** (Section 3). **Third, the share entering the window does not depend on the variable either——0.311640 of Earth's light falls in 8 to 13 μm, with a variable-dependence of $\mathbf{2.109 \times 10^{-15}}$** (Section 2). **Fourth, whether Earth's peak lies in the window depends on the variable——10.061708 μm per wavelength (inside), 17.706399 μm per frequency (outside)** (Section 4). **Fifth, the share calculation matches Paper 222——the Sun's share in 0.4 to 0.7 μm differs by $\mathbf{2.719 \times 10^{-12}}$** (Section 2). **Sixth, the separator is whether a statement compares distributions (shares, crossings) or locates the peak of one distribution——"Earth's light sits in the middle of the atmospheric window" has no truth value until one says whether it means the peak or the share** (Section 5). **Keywords**: atmospheric window, black-body radiation, solar radiation, terrestrial radiation, choice of variable, overlap of distributions.

Scope of this paper (scope note): **No new theorem or law is claimed——Planck's law, Wien's displacement law, the variable-independence of band shares, Earth's radiation escaping readily through the 8 to 13 μm window, and the near absence of overlap between solar and terrestrial radiation are all known (Rybicki and Lightman 1979, Petty 2006).The atmosphere is not computed——8 to 13 μm is given as the band called the window; atmospheric absorption itself is not included.Black bodies——the Sun is an ideal black body at 5772 K and Earth at 288 K; real spectra are not treated.The size of the greenhouse effect is not treated——how much light escapes through the window or how it affects temperature is not stated.Which variable is right is not decided——only what changes and what does not with the choice is written.Relation to earlier papers**: **Paper 222 showed that the Sun peaks in the green, in the orange and in the infrared: one distribution's peak becomes three with the variable, while band shares do not change——its scope note treated a 5772 K ideal black body with no atmosphere and no absorption lines. There one distribution was examined. Here a second distribution (Earth) is set beside it, the crossing and overlap of the two are counted as variable-independent, and "the peak is in the middle of the window" is shown to change with the variable on Earth's side.**What is added is **deriving the crossing wavelength $\mathbf{4.245712}$ μm and the overlap $\mathbf{1.081347} \times 10^{-2}$ of the solar and terrestrial distributions two ways**, **confirming that Earth's window share $\mathbf{0.311640}$ is variable-independent**, **setting Earth's three peaks against the window**, **checking against Paper 222's value**, and **placing the separator on comparison versus peak**.

1. Notation and the Question

Setting: **let $p_T(\lambda)$ be the per-wavelength distribution of black-body light at temperature T, normalized to total 1 ($c_2 = hc/k$). The share in a band is the same whichever variable is integrated.**

$$p_T(\lambda) = \frac{15}{\pi^4}\,\frac{c_2^4}{\lambda^5 T^4\left(e^{c_2/\lambda T}-1\right)}, \qquad \int_{\lambda_1}^{\lambda_2} p_T(\lambda)\,d\lambda = \frac{15}{\pi^4}\int_{x_2}^{x_1}\frac{x^3}{e^x-1}\,dx, \qquad x = \frac{c_2}{\lambda T}$$

Moving the two distributions p_S (Sun) and p_E (Earth) to another variable multiplies both by the same factor. So the crossing point $p_S = p_E$ and the overlap $\int \min(p_S, p_E)$ do not depend on the variable. The peak positions do.

Question: **where do incoming and outgoing light part, and does Earth's light sit in the middle of the window?**

2. Shares in a Band

Computation 2.1: **shares in bands were derived two ways, in the frequency variable and in the wavelength variable.**

quantity	frequency variable	wavelength variable
Sun, 0.4 to 0.7 μm	0.366383171384	0.366383171384
Earth, 8 to 13 μm	0.311640	0.311640
Sun, 8 to 13 μm	1.042343×10^{-3}	—
total (check)	1.000000000002	—

Observation 2.2

the share entering the window does not depend on the variable.0.311640 of Earth's light falls in 8 to 13 μm, with a difference of 2.109×10^{-15} between the two variables. Only 1.042343×10^{-3} of sunlight falls in the same band.

Observation 2.3

the share calculation matches Paper 222.The share of the 5772 K Sun in 0.4 to 0.7 μm differs from Paper 222's 0.36638317138174 by 2.719×10^{-12}; taking 5778 K widens the difference to 2.763×10^{-4}.

3. Dividing Line and Overlap

Computation 3.1: **the wavelength where the two normalized distributions cross was found by bisection, and the overlap derived two ways: as the sum of the two tails cut at the crossing, and as the integral of the smaller distribution.**

quantity	value
crossing wavelength	4.245712 μm
share of sunlight longer than that	8.279013×10^{-3}
share of Earth's light shorter than that	2.534457×10^{-3}
overlap (sum of tails)	1.081347×10^{-2}
overlap (integral of the smaller)	1.081347×10^{-2}
relative difference	5.122×10^{-9}
comparison: cut at 4 μm, Sun long side and Earth short side	9.759215×10^{-3}, 1.448267×10^{-3}

Observation 3.2

this is the core: incoming and outgoing light part at a single wavelength.The two normalized distributions cross at 4.245712 μm, and their overlap is only 1.081347×10^{-2}——over 99 percent of sunlight is shorter than this wavelength, and nearly all of Earth's light is longer. Incoming and outgoing light pass through separate bands.

Observation 3.3

this dividing line and overlap do not depend on the choice of variable.The same change of variable applies to both distributions, so neither the crossing nor the integral of the smaller one changes. The two derivations of the overlap agree to a relative difference of 5.122×10^{-9}——"they part at around 4 μm" compares distributions, so it is the same claim in any variable.

4. Peak Positions

Computation 4.1: **the peak of Earth's light at 288 K was derived in three variables; the per-wavelength peak also matched a numerical search for the maximum.**

how the peak is taken	wavelength	inside the 8 to 13 μm window
per wavelength	**10.061708** μm	inside
per logarithm	**12.742024** μm	inside
per frequency	**17.706399** μm	outside

Observation 4.2

whether Earth's peak lies in the window depends on the variable.The per-wavelength peak at 10.061708 μm is close to the window's middle, but the per-frequency peak at 17.706399 μm lies outside the window ——"the peak sits in the middle of the window" is a claim about one distribution's peak, so it is not settled until a variable is chosen. What Paper 222 showed for the Sun happens on Earth's side too.

5. The Separator

Observation 5.1

the separator is whether a statement compares distributions (shares, crossings) or locates the peak of one distribution.As comparisons, the crossing at 4.245712 μm, the overlap 1.081347×10^{-2} and the window share 0.311640 are all variable-independent; as a peak, the answer can be 10.061708 μm or 17.706399 μm ——"Earth's light sits in the middle of the atmospheric window" has no truth value until one says whether it means the peak or the share.

Observation 5.2

beside Paper 222.Paper 222 showed one distribution's peak becoming three with the variable while band shares stayed fixed——here a second distribution is set beside it, and the crossing and overlap of the two are counted on the variable-independent side.

Observation 5.3

what is not said.Atmospheric absorption itself, real spectra and the size of the greenhouse effect are not treated——what is shown is that the parting of incoming and outgoing light can be stated in variable-independent terms.

6. Roll Call and Closing

1. **crossing wavelength $\mathbf{4.245712}$ µm** (Computation 3.1): **the dividing line.**
2. **overlap $\mathbf{1.081347 \times 10^{-2}}$** (Observation 3.2): **the core.**
3. **relative difference 5.122×10^{-9}** (Observation 3.3): **variable-independent.**
4. **Sun's long side $\mathbf{8.279013 \times 10^{-3}}$** (Computation 3.1): **slight.**
5. **Earth's short side $\mathbf{2.534457 \times 10^{-3}}$** (Computation 3.1): **slight.**
6. **window share $\mathbf{0.311640}$** (Observation 2.2): **variable-independent.**
7. **per-wavelength peak $\mathbf{10.061708}$ µm** (Computation 4.1): **inside the window.**
8. **per-frequency peak $\mathbf{17.706399}$ µm** (Observation 4.2): **outside the window.**
9. **difference from Paper 222, 2.719×10^{-12}** (Observation 2.3): **checking the shares.**
10. **comparison or peak** (Observation 5.1): **the separator; Paper 222.**

Closing: **Incoming and outgoing light part at 4.245712 µm, and this dividing line does not depend on the choice of variable.The normalized light distributions of the Sun at 5772 K and Earth at 288 K cross at 4.245712 µm with an overlap of only 1.081347×10^{-2}, and two derivations agree to a relative difference of 5.122×10^{-9}——the same change of variable applies to both, so neither crossing nor overlap changes.The share 0.311640 of Earth's light in the 8 to 13 µm window is also variable-independent, but Earth's peak is at 10.061708 µm per wavelength (inside the window) and 17.706399 µm per frequency (outside)——"the peak sits in the middle of the window" is not settled until a variable is chosen.The separator is whether a statement compares distributions (shares, crossings) or locates the peak of one distribution.Placed among the earlier papers——Paper 222 separated the peak and the share of one distribution; this counts the dividing line of two distributions on the variable-independent side.To be honest——neither atmospheric absorption, nor real spectra, nor the size of the greenhouse effect is treated.**

References

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