What a Power System’s Inertia Sets Is How Deep the Frequency Sinks, Not Where It Settles

— quadrupling inertia makes the frequency nadir only **1.745117** times shallower, and all cases settle at **−0.238095** Hz, set only by governor droop and load frequency sensitivity — the separator is whether one asks about the nadir on the way or the settling point — [Paper 789]

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— From thinking to syncing. —

Abstract

It is said that the smaller a power system’s inertia (the weight of its spinning generators), the further the frequency drops when generation trips.With a simple one-area model, the initial fall, the nadir on the way and the settling point after a trip are counted while inertia is varied.No new theorem or law is claimed. First, the initial fall is inversely proportional to inertia——−1.250000, −0.625000 and −0.312500 Hz per second for inertia constants of 2, 4 and 8 s (Section 2). **Second, the nadir on the way is deeper the smaller the inertia——−0.746571, −0.563236 and −0.427806 Hz** (Section 2). **Third, yet the nadir is not inversely proportional to inertia——quadrupling inertia makes it only 1.745117 times shallower** (Section 2). **Fourth, and this is the core. The settling point does not depend on inertia——all three settle at −0.238095 Hz, set only by governor droop and load frequency sensitivity** (Section 2). **Fifth, what sets the settling point is the governors——without governors the frequency falls to −5.000000 Hz** (Section 2). **Sixth, the separator is whether one asks about the nadir on the way or the settling point——“a low-inertia system drops its frequency a lot” has no truth value until one says which is meant** (Section 3). **Keywords**: power systems, frequency, inertia constant, governor control, frequency nadir, generation trip.

Scope of this paper (scope note): **No new theorem or law is claimed——the swing equation of generators, the inertia constant, the initial rate of change of frequency being inversely proportional to inertia, and governor droop setting the settled frequency are all known (Kundur 1994, Machowski et al. 2008).One area only——inter-area oscillation, voltage, protection and load shedding are left out.Values are representative——a 10 % trip, load frequency sensitivity 1, droop 0.05 and governor time constant 2 s are for comparison; figures for real systems are not claimed.Governors as a single lag——detailed turbine and control forms are not included.Choice of generation is not discussed——which sources should be added is not stated.Relation to earlier papers**: **Paper 356 showed that the current flows and does no work——its scope note stated that transients were not treated and only steady sinusoids were considered. There, within the steady state, current that flows was separated from power that works. Here the moment after a trip breaks the steady state is followed, separating the nadir from the settling point.Paper 783 showed that where the tides end up is set by angular momentum alone, and how long it takes is set by dissipation——the same type, in which different quantities set the destination and the way there, appears here in a power system.**What is added is **tabulating initial fall, nadir and settling point for three inertias and showing that the nadir is not inversely proportional to inertia (1.745117 times)**, **deriving the nadir two ways, by numerical integration and by a closed form using the matrix exponential**, **showing that the settling point -0.238095 Hz is independent of inertia and becomes -5.000000 Hz without governors**, and **placing the separator on nadir versus settling point**.

1. Notation and the Question

Setting: **write the frequency deviation Δf and the governor's added output ΔP_g as fractions of rating. When generation ΔP_L trips, with inertia constant H, load frequency sensitivity D, droop R and governor time constant T_g , they move as follows.**

$$2H\,\frac{d\,\Delta f}{dt} = \Delta P_g - \Delta P_L - D\,\Delta f, \qquad T_g\,\frac{d\,\Delta P_g}{dt} = -\frac{\Delta f}{R} - \Delta P_g$$

$$\left.\frac{d\,\Delta f}{dt}\right|_{t=0} = -\frac{\Delta P_L}{2H}, \qquad \Delta f_\infty = -\frac{\Delta P_L}{D + 1/R}$$

Take 50 Hz, $\Delta P_L = 0.1$, $D = 1$, $R = 0.05$ and $T_g = 2$ s.

Question: **how far does the frequency drop in a low-inertia system?**

2. Fall, Nadir and Settling Point

Computation 2.1: **for three inertia constants, frequency was followed two ways, by RK4 numerical integration and by a closed form using the matrix exponential; the nadirs from the two differ by at most 4.077×10^{-14} Hz.**

inertia constant	initial fall (formula, numerical)	nadir (time)	time below 49.8 Hz	settling point
2 s	−1.250000, −1.249843 Hz/s	−0.746571 Hz (1.047 s)	0.166 s	−0.238095 Hz
4 s	−0.625000, −0.624961 Hz/s	−0.563236 Hz (1.578 s)	0.335 s	−0.238095 Hz
8 s	−0.312500, −0.312490 Hz/s	−0.427806 Hz (2.437 s)	0.685 s	−0.238095 Hz

Comparison: without governors (inertia 2 s), −4.999773 Hz after 40 s, settling point (formula) −5.000000 Hz.

Observation 2.2

the initial fall is inversely proportional to inertia.Raising inertia from 2 to 8 s cuts the initial fall from −1.250000 to −0.312500 Hz per second, a quarter, and delays crossing 49.8 Hz from 0.166 to 0.685 s.

Observation 2.3

the nadir changes depth but is not inversely proportional to inertia.The nadirs are −0.746571, −0.563236 and −0.427806 Hz, and quadrupling inertia makes the nadir only 1.745117 times shallower——while the fall is slow the governors catch up, so both inertia and governor speed shape the nadir.

Observation 2.4

this is the core: the settling point does not depend on inertia.For all three inertias the settling point is −0.238095 Hz, and inertia does not appear in the formula $-\Delta P_L/(D+1/R)$——inertia sets the depth of the nadir and the speed of the fall; where the frequency settles is set by governor droop and load characteristics.

Observation 2.5

what sets the settling point is the governors.Without governors the settling point is −5.000000 Hz, and after 40 s the frequency has already reached −4.999773 Hz——inertia only slows this fall; it does not stop it.

3. The Separator

Observation 3.1

the separator is whether one asks about the nadir on the way or the settling point.The nadir is deeper for smaller inertia (-0.746571 against -0.427806 Hz); the settling point is -0.238095 Hz regardless of inertia ——"a low-inertia system drops its frequency a lot" has no truth value until one says which is meant.

Observation 3.2

beside Paper 356 and Paper 783.Paper 356 separated flowing current from working power within the steady sinusoid; Paper 783 showed different quantities setting where the tides end up and how long they take—— here the moment after the steady state breaks is followed, and the same type as 783, with different quantities setting the destination and the way there, is counted in a power system.

Observation 3.3

what is not said.Inter-area oscillation, voltage, protection, load shedding, real system figures and the choice of generation are not treated——what is shown is that inertia sets the way there and not the destination.

4. Roll Call and Closing

1. **initial fall -1.250000 Hz per second** (Computation 2.1): **inertia 2 s.**
2. **initial fall -0.312500 Hz per second** (Observation 2.2): **a quarter at 8 s.**
3. **nadir -0.746571 Hz** (Computation 2.1): **inertia 2 s.**
4. **nadir -0.427806 Hz** (Computation 2.1): **inertia 8 s.**
5. **nadir ratio 1.745117** (Observation 2.3): **not inversely proportional.**
6. **settling point -0.238095 Hz** (Observation 2.4): **the core.**
7. **-5.000000 Hz without governors** (Observation 2.5): **governors decide.**
8. **below 49.8 Hz at 0.166 and 0.685 s** (Observation 2.2): **time margin.**
9. **nadir difference 4.077×10^{-14} Hz** (Computation 2.1): **numerical and formula agree.**
10. **nadir or settling point** (Observation 3.1): **the separator; Paper 356 and Paper 783.**

Closing: **What a power system's inertia sets is how deep the frequency sinks, not where it settles.When 10 % of generation trips in a 50 Hz system, inertia constants of 2, 4 and 8 s give initial falls of −1.250000, −0.625000 and −0.312500 Hz per second and nadirs of −0.746571, −0.563236 and −0.427806 Hz——the fall is inversely proportional to inertia, but quadrupling inertia makes the nadir only 1.745117 times shallower.All three settle at −0.238095 Hz, set only by governor droop and load frequency sensitivity, and without governors the frequency falls to −5.000000 Hz——inertia sets the way there, not the destination.The separator is whether one asks about the nadir on the way or the settling point.Placed among the earlier papers——Paper 356 separated current and power within the steady state; this follows the moment the steady state breaks and counts the same type as Paper 783 in a power system.To be honest——it is a simple one-area model, and neither voltage, nor protection, nor real system figures are treated.**

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