Excitation Spreads at the Square Root of Coupling Along a Continuous Line but Stops Partway Along Weakly Coupled Nodes

— with representative values, reaching **25** m/s takes an unmyelinated diameter of **500** μm but a myelinated one of **4.166667** μm, a cross-section ratio of **14400** — the separator is whether the path carrying excitation is continuous or discrete (whether there is myelin) — [Paper 793]

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— From thinking to syncing. —

Abstract

Nerve impulses travel faster in thicker fibres. It is known that fibres without myelin speed up as the square root of diameter, and myelinated fibres in proportion to diameter.Excitation along a continuous line and along discrete nodes is written with one bistable equation, and speed, and the "stopping" that occurs only in the discrete case, are counted.No new theorem or law is claimed. First, the thickness needed for the same speed differs by orders of magnitude with the form of the path——with representative values, 25 m/s needs 500 μm unmyelinated and 4.166667 μm myelinated, a cross-section ratio of 14400 (Section 2). **Second, with strong coupling, discrete nodes carry excitation as fast as a continuous line——0.996181 times the continuous formula at $g = 3$** (Section 3). **Third, with weak coupling, discrete nodes are slower than the formula——0.713506 times at $g = 0.05$** (Section 3). **Fourth, and this is the core. Along discrete nodes, weak coupling stops the excitation——at $g = 0.018$ it does not move on to the next node even after time 3000; at $g = 0.019$ it spreads to the end** (Section 4). **Fifth, the stopping is not due to the initial setup or the time step——halving the time step, or exciting 10 instead of 5 nodes at the start, $g = 0.018$ still stops** (Section 4). **Sixth, the separator is whether the path carrying excitation is continuous or discrete (whether there is myelin)——"weak connections only slow things down" has no truth value until one says whether the path is continuous or discrete** (Section 5). **Keywords**: nerve conduction velocity, myelinated nerves, saltatory conduction, bistable chain, propagation failure, reaction-diffusion.

Scope of this paper (scope note): **No new theorem or law is claimed——unmyelinated fibre speed scaling as the square root of diameter and myelinated fibre speed in proportion to it, the travelling-wave speed $\sqrt{D/2}(1-2a)$ of the bistable reaction--diffusion equation, and propagation failure in chains of coupled discrete cells when coupling is weak are all known (Rushton 1951, Keener 1987).The model is dimensionless——an abstract chain with the bistable reaction $u(1-u)(u-a)$ and $a=0.25$; real membrane currents of nerves are not included.The link to diameter is an assumption——it concerns scaling when diffusion strength is taken proportional to diameter along a continuous line, and node spacing proportional to diameter with coupling unchanged along discrete nodes.Speed coefficients are representative——25 m/s at 500 µm unmyelinated and 6 m/s per µm myelinated are taken as order-of-magnitude values from the literature.Disease is not treated——what happens when myelin is damaged is not stated.Relation to earlier papers**: **Paper 272 showed that one and the same "twice as strong" opens by a factor of 6.70 on the stimulus side——its scope note stated that neural mechanisms, receptors and coding were not entered. There the correspondence between felt and stimulus magnitude was counted. Here, one step earlier, the speed at which excitation travels along a fibre is counted as set by the form of the path, and a discrete path is shown to be able to stop it.**What is added is **setting speeds of the two scalings side by side with representative values and deriving the cross-section ratio 14400 for 25 m/s, deriving speeds numerically in a bistable chain and showing they approach the continuous formula with strong coupling, bracketing the stopping boundary between $g=0.018$ and 0.019 and confirming it with a different time step and initial setup**, and **placing the separator on continuous versus discrete paths**.

1. Notation and the Question

Setting: **let u be the degree of excitation and $f(u)=u(1-u)(u-a)$ the bistable reaction. Along a continuous line excitation spreads with diffusion strength D; along discrete nodes, with coupling g to neighbours.**

$$\frac{\partial u}{\partial t}=D\,\frac{\partial^2 u}{\partial x^2}+f(u),\qquad c=\sqrt{\frac{D}{2}}\,(1-2a)$$

$$\frac{du_n}{dt}=g\,(u_{n+1}-2u_n+u_{n-1})+f(u_n)$$

The continuous speed c is proportional to $\sqrt{D}$. A discrete chain with strong coupling is close to $\sqrt{g/2}(1-2a)$ nodes per unit time. Take $a=0.25$.

Question: **how does spreading differ when the path carrying excitation is continuous and when it is discrete?**

2. Thickness and Speed

Computation 2.1: **with representative coefficients (25 m/s at 500 µm unmyelinated; 6 m/s per µm myelinated), speeds at several diameters and the diameters needed for 25 m/s were derived.**

diameter	unmyelinated (square root)	myelinated (proportional)	ratio
1 μm	1.118034 m/s	6.000000 m/s	5.366563
5 μm	2.500000 m/s	30.000000 m/s	12.000000
20 μm	5.000000 m/s	120.000000 m/s	24.000000

Diameters needed for 25 m/s: 500.000000 μm unmyelinated, 4.166667 μm myelinated; diameter ratio 120.000000, cross-section ratio 14400.000000.

> **Observation 2.2**
>
> **the thickness needed for the same speed differs by orders of magnitude with the form of the path.Because the exponents are $1/2$ and 1, the ratio grows with diameter, and the cross-section needed for 25 m/s differs 14400 times——thickening and making the path discrete pay different thicknesses for the same speed.**

3. Speed Along Discrete Nodes

Computation 3.1: in a chain of 60 nodes with the first 5 excited, the speed from node 20 to 40 was measured and compared with the continuous formula.

coupling g	speed (nodes per time)	continuous formula	ratio
0.05	0.056408	0.079057	0.713506
0.1	0.096759	0.111803	0.865437
0.3	0.186204	0.193649	0.961553
1.0	0.349664	0.353553	0.988998
3.0	0.610034	0.612372	0.996181

Observation 3.2

with strong coupling, discrete nodes carry excitation as fast as a continuous line.At $g = 3$ the speed is 0.996181 times the formula, at $g = 1$ 0.988998 times——if node spacing is short compared with the spread of excitation, discreteness does not matter.

Observation 3.3

with weak coupling, discrete nodes are slower than the formula.The ratio falls to 0.865437 at $g = 0.1$ and 0.713506 at $g = 0.05$——the continuous formula describes the discrete chain less and less as coupling weakens.

4. The Stopping Boundary

Computation 4.1: **in a chain of 14 nodes with the first 5 excited, the number of excited nodes after time 3000 was counted (time step 0.02).**

coupling g	excited nodes after time 3000	behaviour
0.010	5	stops
0.015	5	stops
0.018	5	stops
0.019	14	spreads to the end
0.020	14	spreads to the end

With time step 0.01, $g = 0.018$ gave 5 and $g = 0.019$ gave 14. At $g = 0.018$, exciting 10 nodes at the start left 10.

Observation 4.2

this is the core: along discrete nodes, weak coupling stops the excitation.The stopping boundary lies between $g = 0.018$ and 0.019; below it, excitation does not move on to the next node even after time 3000——the continuous formula gives a nonzero speed at $g = 0.018$, but in the discrete chain the edge of excitation catches between nodes and stays put.

Observation 4.3

the stopping is not due to the initial setup or the time step.Halving the time step, or exciting 10 nodes at the start, it did not spread below the boundary and did spread above it——watching only a short time can mistake a slowly spreading chain for a stopped one, so counting was done over a long time.

5. The Separator

Observation 5.1

the separator is whether the path carrying excitation is continuous or discrete (whether there is myelin).Along a continuous line, weakening coupling only lowers the speed as a square root; along discrete nodes, $g = 0.018$ stops it——"weak connections only slow things down" has no truth value until one says whether the path is continuous or discrete.

Observation 5.2

beside Paper 272.Paper 272 showed, without entering neural mechanisms, that a felt doubling opens by 6.70 times on the stimulus side——here, one step earlier, the speed of excitation and its stopping boundary are counted as set by the form of the path.

Observation 5.3

what is not said.Real membrane currents, the actual correspondence of diameter and coupling, and diseases of myelin are not treated——what is shown is that a discrete path has a stopping boundary that a continuous path lacks.

6. Roll Call and Closing

1. **500 µm unmyelinated for 25 m/s** (Computation 2.1): **the path of thickening.**
2. **4.166667 µm myelinated** (Computation 2.1): **the path of discreteness.**
3. **cross-section ratio 14400** (Observation 2.2): **orders apart.**
4. **0.996181 at $g = 3$** (Observation 3.2): **as fast as a continuous line.**
5. **0.713506 at $g = 0.05$** (Observation 3.3): **slower than the formula.**
6. **stops at $g = 0.018$** (Computation 4.1): **the core.**
7. **spreads at $g = 0.019$** (Computation 4.1): **above the boundary.**
8. **same with half the time step** (Observation 4.3): **not the step.**
9. **still stops with 10 initial nodes** (Observation 4.3): **not the setup.**
10. **continuous or discrete path** (Observation 5.1): **the separator; Paper 272.**

Closing: **Excitation spreads at the square root of the coupling along a continuous line, but stops partway along discrete nodes when coupling is weak.With representative values, unmyelinated nerves speed up as the square root of diameter and myelinated ones in proportion, so 25 m/s needs 500 µm and 4.166667 µm, a cross-section ratio of 14400——the same speed costs different thickness by the form of the path.In a bistable chain, node speed at $g = 3$ is 0.996181 times the continuous formula and 0.713506 times at $g = 0.05$; at $g = 0.018$ excitation does not move on to the next node even after time 3000, while at $g = 0.019$ it spreads to the end, and halving the step or exciting more nodes at the start kept each case on the same side——a discrete path has a stopping boundary that a continuous path lacks.The separator is whether the path carrying excitation is continuous or discrete (whether there is myelin).Placed among the earlier papers——Paper 272 counted felt versus stimulus magnitude; this counts the speed of excitation and its stopping boundary.To be honest——the model is a dimensionless bistable chain, with neither real nerve membranes nor disease treated.**

References

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