Minimizing Flow Work Plus the Cost of Blood Makes Flow Scale as the Radius Cubed and Keeps Wall Shear Stress at Branches

— the logarithmic slope of flow against optimal radius is **3.000000**, and at an equal split each child's radius is **0.793701** of the parent's with wall shear stress kept at **1.000000** — the separator is what is minimized (flow work plus the cost of keeping blood, or preserved cross-section or flow speed) — [Paper 795]

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— From thinking to syncing. —

Abstract

Blood vessels narrow each time they branch. A wider vessel costs less work to push blood through, but more blood must be kept in the body.The radius minimizing the sum of flow work and the cost of keeping blood is searched numerically, and its relation to flow, the radius, cross-section, speed and wall shear stress at a branch, and the branching angle are counted.No new theorem or law is claimed. First, and this is the core. The cube of the optimal radius is proportional to flow——varying flow from 10^{-4} to 10^{-9} m^3/s gives a logarithmic slope of 3.000000 (Section 2). **Second, splitting into two equal branches makes each child's radius $2^{-1/3}$ of the parent's——0.793701 times, with total cross-section growing to 1.259921 times** (Section 3). **Third, wall shear stress does not change at a branch——child over parent is 1.000000, while flow speed falls to 0.793701 times** (Section 3). **Fourth, an area-preserving split is not optimal——each child's cost is 1.058267 times the optimum and its flow work 1.587401 times** (Section 3). **Fifth, the branching angle follows from the same minimization——37.467311 degrees on each side, 74.934622 degrees in total** (Section 4). **Sixth, the separator is what is minimized (the sum of flow work and the cost of keeping blood, or preservation of cross-section or flow speed)——"vessels narrow at branches so as to preserve the flow" has no truth value until one says what is preserved** (Section 5). **Keywords**: Murray's law, vascular branching, tube flow, minimum work, wall shear stress, branching angle.

Scope of this paper (scope note): **No new theorem or law is claimed——flow proportional to the cube of the radius when the sum of flow work and blood-volume cost is minimized (Murray's law), constant wall shear stress in that case, and branching angles following from the same minimization are all known (Murray 1926, Sherman 1981).The cost is a set value——the cost of keeping blood, b, was set for comparison so that a radius of 0.0125 m is optimal at 10^{-4} m^3/s; it is not measured. The slope of the flow-radius relation and the branching ratios do not depend on b.The flow is simplified——only Newtonian, laminar, steady tube flow; pulsation, turbulence, viscosity changes from red cells and wall elasticity are left out.No claim about real vessels——which vessels in the body follow this form is not stated.Only equal two-way splits——unequal branches or splits into three or more are not entered.Relation to earlier papers**: **Paper 576 showed that widening a water main makes the water in it older——its scope note stated that only one pipe was considered, without mixing in networks or loops. There a single pipe's width set the trade between flow speed and water age. Here, in a branching network of tubes, the trade between flow work and holding cost is counted as setting the cube relation between flow and radius and the branching angle.**What is added is **searching the optimal radius numerically and matching the formula at four flows, with slope $\mathbf{3.000000}$, tabulating radius, cross-section, speed and wall shear at a branch, comparing costs with an area-preserving split, deriving the branching angle two ways, numerically and by formula**, and **placing the separator on what is minimized**.

1. Notation and the Question

Setting: **let μ be blood viscosity, Q flow, r tube radius, and b the cost per volume of keeping blood. The cost per length C is the sum of flow work and the cost of keeping blood.**

$$C(r) = \frac{8\mu Q^2}{\pi r^4} + b\pi r^2, \qquad \frac{dC}{dr} = 0 \implies Q = \pi r^3 \sqrt{\frac{b}{16\mu}}, \qquad \tau = \frac{4\mu Q}{\pi r^3}$$

When a tube splits into two equal branches each carrying half the flow, the optimal child radius satisfies $r_1^3 = r_0^3/2$. τ is the wall shear stress, proportional to Q/r^3. Take $\mu = 3.5 \times 10^{-3}$ Pa s, with $b = 14.874015$ W/m^3 set so that a radius of 0.0125 m is optimal at 10^{-4} m^3/s.

Question: **what sets the widths of a branching network of tubes?**

2. Flow and Radius

Computation 2.1: **for four flows, the radius minimizing C was searched by golden-section and compared with the formula.**

flow	optimal radius (numerical)	formula
1.0×10^{-4} m^3/s	$1.249999998 \times 10^{-2}$ m	$1.250000000 \times 10^{-2}$ m
5.0×10^{-5} m^3/s	$9.921256577 \times 10^{-3}$ m	$9.921256575 \times 10^{-3}$ m
1.0×10^{-6} m^3/s	$2.693043358 \times 10^{-3}$ m	$2.693043363 \times 10^{-3}$ m
1.0×10^{-9} m^3/s	$2.693043357 \times 10^{-4}$ m	$2.693043363 \times 10^{-4}$ m

Observation 2.2

this is the core: the cube of the optimal radius is proportional to flow.Varying flow over five decades from 10^{-4} to 10^{-9} m^3/s, the logarithmic slope of flow against optimal radius is 3.000000——flow work enters as radius to the -4 and holding cost as radius squared, and their balance produces the cube relation.

3. Branching

Computation 3.1: the optimal radius at half the flow was searched and compared with the parent, and also the cost of an area-preserving split (child radius $=$ parent$/\sqrt{2}$).

quantity (child / parent)	value
radius	0.793701 ($2^{-1/3} = 0.793701$)
total cross-section	1.259921
flow speed	0.793701
wall shear stress	1.000000
area-preserving child cost / optimum	1.058267
its flow work / optimal flow work	1.587401

Observation 3.2

splitting into two equal branches makes each child's radius $2^{-1/3}$ of the parent's.The child radius is 0.793701 times, the two children's total cross-section grows to 1.259921 times the parent's, and flow speed falls to 0.793701 times——the more the tubes branch, the wider the total cross-section and the slower the flow.

Observation 3.3

wall shear stress does not change at a branch.The ratio of child to parent shear stress is 1.000000—— because flow is proportional to the cube of the radius, the shear stress, proportional to Q/r^3, is the same in every branch.

Observation 3.4

an area-preserving split is not optimal.Making the child radius parent$/\sqrt{2}$ to keep flow speed gives a child cost 1.058267 times the optimum and flow work 1.587401 times——keeping flow speed and minimizing the cost sum are different demands.

4. The Branching Angle

Computation 4.1: **with the parent tube starting at the origin and two children heading to $(1, \pm 0.5)$, the branch point was placed by golden-section to minimize the total cost, and the half-angle derived. The formula is $\cos\theta = w_0/(2w_1)$, with w the optimal cost per length.**

quantity	value
half-angle (numerical)	**37.467311** degrees
half-angle (formula)	**37.467311** degrees
$\arccos 2^{-1/3}$	**37.467311** degrees
total angle	**74.934622** degrees

Observation 4.2

the branching angle follows from the same minimization.The half-angle, numerical and formula agreeing to 1.950×10^{-7} degrees, is 37.467311 degrees, and the total is 74.934622 degrees——because the optimal cost is proportional to radius squared, the angle is $\arccos 2^{-1/3}$, a number set only by the radius ratio.

5. The Separator

Observation 5.1

the separator is what is minimized (the sum of flow work and the cost of keeping blood, or preservation of cross-section or flow speed).Minimizing the cost sum gives child radius 0.793701 times with shear stress preserved; preserving cross-section keeps flow speed but costs 1.058267 times as much——"vessels narrow at branches so as to preserve the flow" has no truth value until one says what is preserved.

Observation 5.2

beside Paper 576.Paper 576 showed, for one water main without networks, that width sets the trade between flow speed and water age——here, in a branching network, the trade between flow work and holding cost is counted as setting the cube relation and the angle.

Observation 5.3

what is not said.Neither a measured cost b, nor pulsation or turbulence, nor real vessel dimensions are treated——what is shown is that one minimization yields the cube relation, constant shear stress and the branching angle together.

6. Roll Call and Closing

1. **slope 3.000000** (Observation 2.2): **the core.**
2. **child radius 0.793701 times** (Observation 3.2): **$2^{-1/3}$.**
3. **total cross-section 1.259921 times** (Observation 3.2): **widening.**

4. **flow speed 0.793701 times** (Observation 3.2): **slowing.**
5. **shear stress 1.000000 times** (Observation 3.3): **preserved.**
6. **area-preserving cost 1.058267 times** (Observation 3.4): **not optimal.**
7. **its flow work 1.587401 times** (Observation 3.4): **a different demand.**
8. **half-angle 37.467311 degrees** (Observation 4.2): **the branching angle.**
9. **total 74.934622 degrees** (Computation 4.1): **set by the radius ratio alone.**
10. **what is minimized** (Observation 5.1): **the separator; Paper 576.**

Closing: **Minimizing the work of flow plus the cost of keeping blood makes flow proportional to the cube of the radius and keeps wall shear stress at every branch.Over five decades of flow the logarithmic slope of flow against optimal radius is 3.000000, and an equal two-way split gives child radius 0.793701 times, total cross-section 1.259921 times, flow speed 0.793701 times and shear stress 1.000000 times——the balance of flow work and holding cost produces the cube relation and constant shear stress together.An area-preserving split costs 1.058267 times the optimum, and the same minimization sets the half-angle at 37.467311 degrees——keeping flow speed and minimizing the cost are different demands.The separator is what is minimized (the sum of flow work and the cost of keeping blood, or preservation of cross-section or flow speed).Placed among the earlier papers——Paper 576 counted the width trade of one pipe; this counts the trade in a branching network.To be honest——the cost is a set value, and neither pulsation, turbulence nor real vessel dimensions are treated.**

References

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