Radioactive Equilibrium Itself Gives No Age: Only the Departure From Equilibrium Keeps Time

— At 1, 3, 5, 7 and 10 daughter half-lives the daughter-to-parent activity ratio is 0.5000000, 0.8750000, 0.9687500, 0.9921875 and 0.9990234 — the separator is whether age is read from the departure from equilibrium, and whether that departure exceeds the measurement error — [Paper 796]

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— From thinking to syncing. —

Abstract

When a radioactive parent produces a daughter that also decays, after long enough the daughter and parent activities become equal (radioactive equilibrium). The approach to equilibrium is set by the daughter's half-life, so it can serve as a clock.The approach to equilibrium, the error in age read from the ratio, and cases where the equilibrium ratio is not 1 are counted, to see what actually keeps time.No new theorem or law is claimed. First, equilibrium is approached by halving the gap each half-life——0.9921875 at 7 half-lives, 0.9990234 at 10 (Section 2). **Second, and this is the core. The closer to equilibrium, the more the age error swells——with ratio error ±0.005, the age error is 0.014427 of the age at 1 half-life and 0.131904 at 7** (Section 3). **Third, the clock reads best not at exactly one half-life——the relative error is smallest at $1/\lambda$, 1.442695 half-lives, where it is 0.013591** (Section 3). **Fourth, too close to equilibrium, no age comes out at all——at 7 half-lives the measured ratio exceeds 1 in 0.059150 of cases; ten-percent precision holds only up to 6.491775 half-lives** (Section 3). **Fifth, the equilibrium ratio is not always 1——if the parent's half-life is only about 11 times the daughter's, the ratio heads to 1.100223** (Section 4). **Sixth, the separator is whether age is read from the departure from equilibrium, and whether that departure exceeds the measurement error——"it has reached equilibrium, so it is old" does not become an age until one says whether the departure exceeds the error** (Section 5). **Keywords**: radioactive equilibrium, secular equilibrium, transient equilibrium, radiometric dating, error propagation, Bateman equations.

Scope of this paper (scope note): **No new theorem or law is claimed——the coupled decay equations of parent and daughter and their solution (Bateman), secular and transient equilibrium, reading age from the daughter-to-parent activity ratio, and how ratio error propagates to age error are all known (Bateman 1910, Faure and Mensing 2005).Values are representative——a daughter half-life of 75600 years, ratio error ± 0.005, and a transient pair of parent 65.94 hours and daughter 6.0067 hours are for comparison; the precision of particular samples or laboratories is not claimed.No initial daughter——cases with daughter present at the start, or with gains and losses along the way (open systems), are not treated.Two-member chains only——decay series of three or more members and branching decays are left out.Dating practice is not treated——which clock to use for which sample is not stated.Relation to earlier papers**: **Paper 346 showed that for two of the three reaction orders, "half-life" is not even a property of the substance ——its scope note stated that the mechanism of radioactive decay was not entered, nor reversible or sequential reactions. There the decrease of a single substance was examined. Here, in the sequential decay from parent to daughter, the daughter's half-life is counted as a clock that becomes unreadable the closer it is to equilibrium.**What is added is **propagating ratio error to age error and deriving the ten-percent limit of $\mathbf{6.491775}$ half-lives and the best-reading point at $\mathbf{1.442695}$ half-lives**, **counting by random sampling the spread of ages and the share of cases giving no age**, **deriving the transient equilibrium ratio two ways, by formula and numerical integration**, and **placing the separator on reading the departure and on whether it exceeds the error**.

1. Notation and the Question

Setting: **let λ_A be the parent decay constant and λ_B the daughter's. If the parent is very long-lived, the daughter-to-parent activity ratio R approaches 1 as follows, and age can be read from the ratio. Ratio error σ_R propagates to age error σ_t.**

$$R(t) = 1 - e^{-\lambda_B t}, \qquad t = -\frac{\ln(1-R)}{\lambda_B}, \qquad \sigma_t = \frac{\sigma_R}{\lambda_B (1-R)}$$

When the parent is not long-lived, the ratio's destination and the time of maximum daughter activity are as follows (transient equilibrium).

$$\frac{A_B}{A_A} \to \frac{\lambda_B}{\lambda_B - \lambda_A}, \qquad t_{\max} = \frac{\ln(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}$$

Question: **when radioactive equilibrium is used as a clock, what is keeping time?**

2. Approach to Equilibrium

Computation 2.1: **the ratio R was derived at multiples of the daughter half-life.**

elapsed time (daughter half-lives)	activity ratio
1	0.5000000
3	0.8750000
5	0.9687500
7	0.9921875
10	0.9990234

> **Observation 2.2**
>
> equilibrium is approached by halving the gap each half-life.The remaining departure $1 - R$ halves every half-life: 0.9921875 at 7 half-lives, 0.9990234 at 10——the nearer the ratio is to 1, the less the ratio itself says about time.

3. Age Error

Computation 3.1: with a daughter half-life of 75600 years and ratio error ± 0.005, ages and their errors were derived, and the same counted by random sampling (40000 trials).

age	ratio	age error	share of age
1 half-life (75600 yr)	0.500000	1090.677451 yr	0.014427
3 half-lives (226800 yr)	0.875000	4362.709804 yr	0.019236
5 half-lives (378000 yr)	0.968750	17450.839215 yr	0.046166
7 half-lives (529200 yr)	0.992188	69803.356858 yr	0.131904

counted by random sampling	3 half-lives	7 half-lives
spread of ages	**4392.807006** yr	**86021.414825** yr
ratio to the error formula	**1.006899**	**1.232339**
share with ratio above **1** (no age)	**0.000000**	**0.059150**

Observation 3.2

this is the core: the closer to equilibrium, the more the age error swells.The age error is 0.014427 of the age at 1 half-life, 0.046166 at 5 and 0.131904 at 7——what keeps time is not equilibrium itself but the departure $1 - R$; the smaller it gets, the wider the span of ages the same error covers.

Observation 3.3

the clock reads best not at exactly one half-life.The share of error relative to age is smallest at $1/\lambda$: 109067.745091 years by formula (1.442695 half-lives) and 109067.744366 years numerically, with a share of 0.013591 ($\sigma_R e$).

Observation 3.4

too close to equilibrium, no age comes out at all.The age error stays within ten percent only up to 490778.162781 years, 6.491775 half-lives. At 7 half-lives the random spread widens to 1.232339 times the formula, and in 0.059150 of cases the measured ratio exceeds 1 and gives no age——once the departure is comparable to the measurement error, the error formula itself becomes unreliable.

4. When the Equilibrium Ratio Is Not 1

Computation 4.1: **for a pair with parent 65.94 hours and daughter 6.0067 hours, the ratio's destination, the time of maximum daughter activity and the ratio at 200 hours were derived two ways, by formula and numerical integration.**

quantity	value
destination of the ratio	1.100223
time of maximum daughter activity	22.843068 hours
ratio at 200 hours (formula, numerical)	1.100223, 1.100223
difference between the two	2.887×10^{-15}

Observation 4.2

the equilibrium ratio is not always 1.With the parent's half-life only about 11 times the daughter's, the ratio heads not to 1 but to 1.100223——a steady ratio marks equilibrium, but whether its value is 1 depends on how long-lived the parent is.

5. The Separator

Observation 5.1

the separator is whether age is read from the departure from equilibrium, and whether that departure exceeds the measurement error.At 1 half-life, where the departure well exceeds the error, age reads to 0.014427; at 7 half-lives, where the departure is comparable to the error, this swells to 0.131904, and 0.059150 of cases give no age——"it has reached equilibrium, so it is old" does not become an age until one says whether the departure exceeds the error.

Observation 5.2

beside Paper 346.Paper 346 split by reaction order whether "half-life" can be a property of a single decreasing substance, leaving sequential reactions aside——here, in sequential decay from parent to daughter, the daughter's half-life is counted as a clock that becomes unreadable near equilibrium.

Observation 5.3

what is not said.Initial daughter, open systems, longer series and the precision of real samples or laboratories are not treated——what is shown is that time is kept by the departure, not by equilibrium, and the clock appears to stop as the departure vanishes.

6. Roll Call and Closing

1. **0.9921875 at 7 half-lives** (Computation 2.1): **near equilibrium.**
2. **error 0.014427 at 1 half-life** (Computation 3.1): **reads well.**
3. **error 0.131904 at 7 half-lives** (Observation 3.2): **the core.**
4. **minimum at 1.442695 half-lives** (Observation 3.3): $1/\lambda$**.**
5. **minimum share 0.013591** (Observation 3.3): $\sigma_R e$**.**
6. **ten-percent limit 6.491775 half-lives** (Observation 3.4): **the readable range.**
7. **share giving no age 0.059150** (Observation 3.4): **departure comparable to error.**
8. **random-to-formula ratio 1.232339** (Computation 3.1): **the error formula breaks down.**
9. **transient ratio 1.100223** (Observation 4.2): **not 1.**
10. **reading the departure, and whether it exceeds the error** (Observation 5.1): **the separator; Paper 346.**

Closing: **Radioactive equilibrium itself gives no age: only the departure from equilibrium keeps time.The departure halves each daughter half-life, so the ratio is 0.9921875 at 7 half-lives and 0.9990234 at 10——the nearer the ratio is to 1, the less it says about time.Reading with ratio error ±0.005, the age error is 0.014427 of the age at 1 half-life and swells to 0.131904 at 7; it reads best at 1.442695 half-lives (0.013591), stays within ten percent up to 6.491775 half-lives, and at 7 half-lives 0.059150 of cases give no age. If the parent is not long-lived enough, the equilibrium ratio heads to 1.100223——time is kept by the departure, not by equilibrium, and when the departure is comparable to the error the clock appears to stop.The separator is whether age is read from the departure from equilibrium, and whether that departure exceeds the measurement error.Placed among the earlier papers——Paper 346 split the half-life of one substance by reaction order; this counts the readable range of the daughter-half-life clock in sequential decay.To be honest——values are representative, and neither open systems, long series, nor real samples are treated.**

References

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