Three Explanations of the Missing Fundamental Split into Jumping, Standing Still and Moving Smoothly Under an Equal Shift

— a sound of **400**, **600** and **800** Hz has no **200** Hz component, yet the greatest common divisor, the difference tone and the first autocorrelation peak all return **200** Hz — the separator is whether, when the components are shifted equally, the answer jumps, stands still or moves smoothly — [Paper 801]

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— From thinking to syncing. —

Abstract

Sounding components at 400, 600 and 800 Hz is said to produce a pitch of 200 Hz that is not there: the missing fundamental.Three explanations offered for it—the greatest common divisor, a difference tone made by distortion, and the autocorrelation of the waveform—are computed in their simplest forms, and an operation that separates them is sought.No new theorem or law is claimed. First, with the three components as they are, all three explanations return 200 Hz——greatest common divisor 200 Hz, differences 200 and 400 Hz, first autocorrelation peak at 5.000000 ms (Section 2). **Second, the waveform itself has no 200 Hz component——3.752×10^{-16} at 200 Hz against 0.500000 at 400 Hz** (Section 2). **Third, and this is the core. Shifting the components equally splits the three answers——a 10 Hz shift gives a greatest common divisor of 10 Hz, a difference tone of 200 Hz, and autocorrelation at 203.106667 Hz** (Section 3). **Fourth, continuing the shift gives jumping, standing still and moving smoothly——the greatest common divisor runs 10, 20, 10, 50 Hz, autocorrelation continues through 206.218462, 209.333585 and 215.567394 Hz, rising by 3.106667, 3.111795 and 3.115123 per 10 Hz** (Section 3). **Fifth, sampled sound and random phases do not change the answers——sampled autocorrelation gives 199.995555 Hz and 203.112115 Hz, and the strongest frequency of the squared distortion is 200 Hz for all 12 phase sets** (Section 4). **Sixth, the separator is whether, when the components are shifted equally, the answer jumps, stands still or moves smoothly——"why the missing fundamental is heard" can be said to be any of the three for an unshifted sound and is not decided until the components are shifted** (Section 5). **Keywords**: missing fundamental, difference tone, autocorrelation, greatest common divisor, overtones, pitch.

Scope of this paper (scope note): **No new theorem or law is claimed——that the pitch of the fundamental is heard from harmonic components, the idea of explaining it by a difference tone made by distortion in the ear (Helmholtz 1863), that the heard pitch moves when the components are shifted equally (Schouten, Ritsma and Cardozo 1962), and the idea of reading the period by autocorrelation (Licklider 1951) are all known.No model of hearing is built——the three explanations are only computed in their simplest forms (the integer greatest common divisor, squaring distortion, the first peak of the waveform's autocorrelation), and no mechanism inside the ear is built (the same line as Paper 672).No value of heard pitch is used——experimentally measured pitches are not used; only the answers the three explanations return are set side by side. Which explanation is right is not decided.The rule for picking a peak is a convention——because the autocorrelation has peaks of equal height at 5 ms and 10 ms, "the shortest lag among peaks at least 0.9 of the maximum" was set as the rule. A different rule gives a different answer.All components have equal amplitude——phases are drawn at random, and differences in amplitude are left out.Instruments are not discussed——which instruments produce sounds of this form is not the subject.Relation to earlier papers**: **Paper 672 showed that what decides between "two tones" and "one wavering tone" is not the sound but the listener, and its scope note stated that no model of hearing was built——this paper stands beside that seat: it builds none either, and splits the listener-side question into three answers computable from the component frequencies.Paper 582 showed that what sets the series of overtones is not the closed end but how the tube widens——there, how the overtone series is made was counted. Here, how a pitch not present is read from a series of overtones is counted.Paper 299 showed that a drum has no pitch because of its boundary, not its equation——there the components did not line up in integer ratios and the pitch was undetermined. Here components lined up in integer ratios are shifted equally to break those ratios.**What is added is **showing that all three explanations return 200 Hz for the unshifted sound**, **counting that under equal shifts the greatest common divisor jumps, the difference tone stands still and autocorrelation moves smoothly from 203.106667 Hz**, **confirming that sampled sound and 12 phase sets do not change the answers**, and **placing the separator on whether the answer jumps, stands still or moves smoothly under a shift**.

1. Notation and the Question

Setting: **consider a sound $x(t)$ made of three components f_1, f_2, f_3 of equal amplitude. The explanations for the pitch of the missing fundamental are written in their three simplest forms.**

$$f_{\text{gcd}} = \gcd(f_1, f_2, f_3), \qquad f_{\text{diff}} \in \{\, |f_i - f_j| \,\}, \qquad f_{\text{ac}} = \frac{1}{\tau_1}, \quad R(\tau) = \frac{1}{3}\sum_{i=1}^{3} \cos 2\pi f_i \tau$$

f_{diff} is the difference frequency made by squaring distortion x^2. τ_1 is the shortest lag among the peaks of $R(\tau)$ whose height is at least 0.9 of the maximum (the rule).

Question: **for which sounds do the three explanations return the same answer, and under which operation do they return different ones?**

2. The Unshifted Sound

Computation 2.1: **for the sound at 400, 600 and 800 Hz, the answers of the three explanations and the components of the waveform itself were derived (waveform sampled at 48000 Hz for 0.5 seconds).**

quantity	value
greatest common divisor	200 Hz
differences (difference tones)	200 and 400 Hz
autocorrelation peaks (height)	5.000000 ms (1.000000), 10.000000 ms (1.000000)
peak chosen by the rule	5.000000 ms ⇒ 200.000000 Hz
200 Hz component in the waveform	3.752×10^{-16}
400 Hz component in the waveform	0.500000

> **Observation 2.2**
>
> **with the three components as they are, all three explanations return 200 Hz.The 200 Hz component of the waveform is at the level of rounding error, so no fundamental is there, yet the greatest common divisor, the difference tone and the autocorrelation all point to 200 Hz——so however long one listens to the unshifted sound, the three explanations cannot be separated.**

> **Observation 2.3**
>
> **the autocorrelation's answer depends on the rule for picking a peak.The peaks at 5 ms and 10 ms have the same height 1.000000, and without a rule the 10 ms peak could equally be read as 100 Hz—— autocorrelation returns not "the period" but a row of candidates, and the rule does the choosing.**

3. Shifting Equally

Computation 3.1: **the three components were shifted equally and the answers of the three explanations derived. Autocorrelation is the first peak chosen by the rule, with its height in parentheses.**

shift	greatest common divisor	differences	first autocorrelation peak	middle component ÷3
0 Hz	200 Hz	200, 400 Hz	200.000000 Hz (1.000000)	200.000000 Hz
10 Hz	10 Hz	200, 400 Hz	203.106667 Hz (0.996703)	203.333333 Hz
20 Hz	20 Hz	200, 400 Hz	206.218462 Hz (0.987241)	206.666667 Hz
30 Hz	10 Hz	200, 400 Hz	209.333585 Hz (0.972256)	210.000000 Hz
50 Hz	50 Hz	200, 400 Hz	215.567394 Hz (0.928236)	216.666667 Hz

Observation 3.2

this is the core: shifting the components equally splits the three answers.The greatest common divisor jumps non-monotonically through 10, 20, 10 and 50 Hz, the difference tone stays at 200 Hz, and autocorrelation rises smoothly through 203.106667, 206.218462, 209.333585 and 215.567394 Hz, by 3.106667, 3.111795 and 3.115123 per 10 Hz——to the single operation of shifting, the three explanations answer with three different behaviours.

Observation 3.3

the autocorrelation answer is near the middle component divided by 3 but does not equal it.With a 10 Hz shift it is 203.106667 Hz against 203.333333 Hz. The peak height falls from 0.996703 to 0.928236, and at a 50 Hz shift the next peaks sit at 9.282702 ms and 10.717298 ms with equal height 0.730278——the more the shift, the smaller the gap between chosen and unchosen peaks, and the more weight the rule carries.

4. Sampled Sound and Random Phases

Computation 4.1: **sounds sampled at 48000 Hz for 0.5 seconds with random phases were analysed for autocorrelation peaks.**

components	autocorrelation peaks (height)	answer by the rule
400, 600, 800 Hz	5.0001 ms (1.0000), 10.0001 ms (1.0000)	199.995555 Hz
410, 610, 810 Hz	4.9234 ms (0.9963), 9.8469 ms (0.9864)	203.112115 Hz

Computation 4.2: **the sound at 410, 610 and 810 Hz was made with 12 phase sets, and the strongest frequency of the squared distortion x^2 between 150 and 260 Hz (1 Hz steps, 1 second) was sought. For all 12 it was 200 Hz, and its strength was at least 5.815×10^{12} times that at 203 Hz.**

> **Observation 4.3**
>
> **sampled sound and random phases do not change the answers.Sampled autocorrelation differs from the formula's peaks by only -0.004445 Hz and $+0.005449$ Hz, and the difference tone is 200 Hz regardless of phase——the way the three answers split is not a product of how the computation was done or which phases were chosen.**

5. The Separator

> **Observation 5.1**
>
> **the separator is whether, when the components are shifted equally, the answer jumps, stands still or moves smoothly.For the unshifted sound all three give 200 Hz; a 10 Hz shift splits them into 10 Hz, 200 Hz and 203.106667 Hz——"why the missing fundamental is heard" can be said to be any of the three for the unshifted sound and is not decided until the components are shifted. That the heard pitch moves under an equal shift is known, but its value is not used here.**

> **Observation 5.2**
>
> **beside Papers 672, 582 and 299.Paper 672 showed that the listener decides between two tones and one, 582 what sets the series of overtones, and 299 that components not in integer ratios have no pitch——here the pitch read from components in integer ratios was split into three explanations by breaking those ratios with a shift.**

> **Observation 5.3**
>
> **what is not said.Mechanisms inside the ear, experimentally measured pitches, differences in component amplitude, instruments, and which explanation is right are not treated——what is shown is that one operation splits three explanations into three behaviours.**

6. Roll Call and Closing

1. **all three give 200 Hz for the unshifted sound** (Observation 2.2): **inseparable.**
2. **first autocorrelation peak at 5.000000 ms** (Computation 2.1): **an equal peak at 10 ms.**
3. **400 Hz component 0.500000** (Computation 2.1): **200 Hz at rounding level.**
4. **greatest common divisor 10 Hz at a 10 Hz shift** (Observation 3.2): **jumps.**
5. **difference tone 200 Hz** (Observation 3.2): **stands still.**
6. **autocorrelation 203.106667 Hz** (Observation 3.2): **the core; moves smoothly.**
7. **215.567394 Hz at a 50 Hz shift** (Computation 3.1): **keeps rising.**
8. **rise per 10 Hz, 3.106667** (Observation 3.2): **nearly constant.**
9. **sampled, 203.112115 Hz** (Computation 4.1): **independent of the method.**
10. **jumps, stands still or moves smoothly** (Observation 5.1): **the separator; Papers 672, 582, 299.**

Closing: **The three explanations for hearing a missing fundamental split into jumping, standing still and moving smoothly when all components are shifted equally.The waveform of the sound at 400, 600 and 800 Hz has no 200 Hz component, yet the greatest common divisor, the difference tone and the first autocorrelation peak all return 200 Hz, so the unshifted sound cannot separate the three——autocorrelation has peaks of equal height at 5 ms and 10 ms, and the rule does the choosing.Shifting the three components equally, the greatest common divisor jumps through 10 , 20, 10 and 50 Hz, the difference tone stays at 200 Hz, and autocorrelation rises smoothly from 203.106667 Hz to 215.567394 Hz, a split unchanged by sampling and by 12 phase sets——to one operation, three explanations answer with three behaviours.The separator is whether, when the components are shifted equally, the answer jumps, stands still or moves smoothly.Placed among the earlier papers——Paper 672 placed the distinction on the listener's side, Paper 582 counted the series of overtones, and Paper 299 components with no pitch. This splits the pitch read from a series of overtones into three explanations.To be honest——no model of hearing is built, and neither measured pitch values nor which explanation is right are stated.**

References

1. Helmholtz, H. von (1863). *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik*. Vieweg.
2. Licklider, J. C. R. (1951). A duplex theory of pitch perception. *Experientia*.
3. Schouten, J. F., Ritsma, R. J., & Cardozo, B. L. (1962). Pitch of the residue. *The Journal of the Acoustical Society of America*.
4. Yamagishi, Y. (2026). *A Drum Has No Pitch Because of Its Boundary, Not Its Equation* [Paper 299]. Zenodo. 10.5281/zenodo.22171877

5. Yamagishi, Y. (2026). *What Sets the Series of Overtones Is Not the Closed End but How the Tube Widens from It* [Paper 582]. Zenodo. 10.5281/zenodo.23014349

6. Yamagishi, Y. (2026). *What Decides Between "Two Tones" and "One Wavering Tone" Is Not the Sound but the Listener* [Paper 672]. Zenodo. 10.5281/zenodo.23014751

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