# Radar Range Resolution Comes from Bandwidth, Velocity Resolution from Time, and a One-Wrap Velocity Error Shifts Range One Cell

— with $1$ GHz of bandwidth the range resolution is $c/2B = 0.149896$ m, and $30$ m/s wrapping to $-8.935373$ m/s shifts the corrected range by $1.000033$ range resolutions — the separator is whether the velocity has been mistaken by one wrap — [Paper 802]

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— *From thinking to syncing.* —

**Abstract**

**A continuous-wave radar that sweeps its frequency measures a target's range and velocity with one device.What sets the range resolution, the velocity resolution and the velocity limit is laid out, and it is counted how range and velocity mix within one chirp and what happens to the range reading when the velocity exceeds its limit.No new theorem or law is claimed. First, range resolution is set by bandwidth alone, and velocity resolution by observation time——range $0.149896$ m, velocity $0.304173$ m/s** (Section 2). **Second, the ratio of velocity limit to velocity resolution is set only by half the number of chirps——limit $\pm 19.467043$ m/s, ratio $64.000000$** (Section 2). **Third, within one chirp, velocity turns into range——$0.003850$ m per m/s; a 5 m target at 10 m/s appears at $5.038500$ m** (Section 3). **Fourth, reading velocity from the phase across chirps undoes the shift——reading $9.996326$ m/s and correcting gives $5.000014$ m** (Section 3). **Fifth, and this is the core. Mistaking the velocity by one wrap moves the range by exactly one cell——30 m/s wraps to $-8.935373$ m/s, the range corrected with it is $5.149901$ m, off by $1.000033$ range resolutions** (Section 4). **Sixth, the separator is whether the velocity has been mistaken by one wrap ——"the range shift within one chirp can be corrected with the read velocity" is not decided true or false until one says whether the velocity has been mistaken by one wrap** (Section 5). **Keywords**: frequency-modulated continuous-wave radar, range resolution, velocity resolution, Doppler effect, aliasing, observation time.

**Scope of this paper (scope note)**: **No new theorem or law is claimed——the range resolution $c/2B$ of a frequency-modulated continuous-wave radar, the round-trip Doppler effect $2v/\lambda$, reading velocity from the phase across chirps, and the wrapping of velocity readings are all standard (Skolnik's handbook, the textbook by Richards et al.).No circuits or noise are built——mixers, filters, sampling implementation and receiver noise are left out (the same line as Paper 335).A single target moving at constant velocity as a point——extended targets, acceleration and separation of several targets are not treated.The numbers are examples——a centre frequency of 77 GHz, 1 GHz bandwidth, 50-microsecond chirps, 256 samples per chirp and 128 chirps are values for comparison.Methods for resolving the wrap are not treated——procedures such as combining chirps of different lengths are not the subject.No relativistic correction is included——the target speed is taken as small enough against the speed of light, and only the first-order round-trip Doppler formula is used.No device is designed——neither weapons nor products are designed.Relation to earlier papers**: **Paper 335 showed that the round trip changes the exponent of received strength, and its scope note stated that noise and propagation were not treated——this paper stands beside that seat: here the same round trip attaches a 2 to both range and velocity.Paper 491 showed that the classical Doppler effect is two formulas and relativity their geometric mean——here, with target speeds small against light, the difference between the formulas does not appear and the first-order round-trip formula suffices.Paper 649 showed that separating two tidal constituents requires a record as long as their beat period——velocity resolution set by observation time is the same type; here a difference in velocity becomes a difference in how the phase turns from chirp to chirp.Paper 266 showed that the sampling theorem is correct but its premise is never met——the wrapping of velocity is the aliasing of sampling, arising from picking up the phase once per chirp.**What is added is **showing that the ratio of velocity limit to resolution is set only by half the number of chirps, 64.000000, confirming numerically that the range shift within one chirp can be corrected with the velocity, showing by formula that correcting with a wrapped velocity shifts the range by exactly $c/2B$ and confirming it numerically as 1.000033 resolutions**, and **placing the separator on whether the velocity has been mistaken by one wrap**.

## 1. Notation and the Question

**Setting**: **a wave whose frequency is swept linearly through bandwidth $B$ over time $T$ is transmitted, and the returning wave is mixed with it. For a target at range $R$ and velocity $v$, the beat frequency $f_b$ and the phase $\phi_m$ of the $m$-th chirp take the following forms.**

$$f_b = \frac{2SR}{c} + \frac{2v}{\lambda}, \qquad S = \frac{B}{T}, \qquad \phi_m = \frac{4\pi\,(R + v\,mT)}{\lambda}$$

**Range is read from the frequency within one chirp, and velocity from how the phase turns across chirps. With $M$ chirps, resolutions and limit take the following forms.**

$$\Delta R = \frac{c}{2B}, \qquad \Delta v = \frac{\lambda}{2MT}, \qquad v_{\max} = \pm\frac{\lambda}{4T}$$

**Question**: **when range and velocity are read with one device, where do they mix, and what happens to range when velocity is misread?**

## 2. Resolutions and Limit

**Computation 2.1**: **take a centre frequency of 77 GHz, 1 GHz bandwidth, 50-microsecond chirps, 256 samples per chirp and 128 chirps.**

| quantity | value |
|---|---|
| wavelength | **3.893409** mm |
| round-trip Doppler per **1** m/s, $2/\lambda$ | **513.688707** Hz |
| range resolution $c/2B$ | **0.149896** m |
| velocity resolution $\lambda/(2MT)$ | **0.304173** m/s |
| velocity limit $\lambda/(4T)$ | **±19.467043** m/s |
| limit / resolution | **64.000000** |

**Computation 2.2**: **only the chirp length was varied (128 chirps and 1 GHz bandwidth kept).**

| chirp length | velocity limit | velocity resolution | range shift per 1 m/s | range resolution |
|---|---|---|---|---|
| **25** microseconds | **±38.934085** m/s | **0.608345** m/s | **0.001925** m | **0.149896** m |
| **50** microseconds | **±19.467043** m/s | **0.304173** m/s | **0.003850** m | **0.149896** m |
| **100** microseconds | **±9.733521** m/s | **0.152086** m/s | **0.007700** m | **0.149896** m |

> **Observation 2.3**
>
> **range resolution is set by bandwidth alone, velocity resolution by observation time, and the velocity limit by the length of one chirp.Shortening the chirp raises the limit, but with the same number of chirps the observation time shrinks and the resolution coarsens, so the ratio of limit to resolution stays at half the number of chirps, 64.000000——limit and resolution are bought one at the cost of the other.**

$$\frac{v_{\max}}{\Delta v} = \frac{M}{2}$$

## 3. Mixing Within One Chirp

**Computation 3.1**: **for a target at $5$ m, the range read from the frequency of one chirp, the velocity read from the phase over $128$ chirps, and the range corrected with that velocity were derived numerically (grid steps $0.000500$ m in range and $0.009734$ m/s in velocity).**

| true velocity | range from one chirp | velocity from phase | corrected range |
|---|---|---|---|
| $0$ m/s | $5.000000$ m | $0.000000$ m/s | $5.000000$ m |
| $10$ m/s | $5.038500$ m | $9.996326$ m/s | $5.000014$ m |
| $30$ m/s | $5.115500$ m | $-8.935373$ m/s | $5.149901$ m |

> **Observation 3.2**
>
> **within one chirp, velocity turns into range.Because the Doppler term adds to the beat frequency, $1$ m/s turns into $0.003850$ m of range ($0.025684$ range resolutions), and at $10$ m/s the target appears $0.038500$ m farther ——from the frequency of one chirp alone, range and velocity cannot be separated.**

$$\delta R = \frac{v\, f_0\, T}{B}$$

> **Observation 3.3**
>
> **reading velocity from the phase across chirps undoes the shift.$10$ m/s is read as $9.996326$ m/s, and correcting with it gives a range of $5.000014$ m, back within the grid step——range and velocity mix within one chirp but separate across chirps.**

## 4. Mistaking the Velocity by One Wrap

**Computation 4.1**: **$30$ m/s exceeds the velocity limit of $19.467043$ m/s. The velocity read from the phase was $-8.935373$ m/s (formula $30 - \lambda/(2T) = -8.934085$ m/s), the range corrected with it $5.149901$ m, and its gap from the true range $1.000033$ range resolutions (against a sum of grid steps of $0.000537$ m, the measured difference was $+0.000005$ m).**

**Observation 4.2**

**this is the core: mistaking the velocity by one wrap moves the range by exactly one cell.The velocity reading wraps every $\lambda/(2T)$, so the mistake is one wrap, $\lambda/(2T)$, and the range it turns into within one chirp is, by the following formula, $c/2B$ itself (the formula gives $0.149896229$ m, and $c/2B$ is $0.149896229$ m)——misreading the velocity by one wrap moves the supposedly corrected range exactly into the neighbouring cell.**

$$\frac{\lambda}{2T}\cdot\frac{f_0 T}{B}=\frac{c}{2B}=\Delta R$$

**Observation 4.3**

**the size of one cell does not depend on chirp length.In the formula above the chirp length $T$ cancels, and the remaining $c/2B$ is the range resolution itself——whether chirps are shortened or lengthened, the range error from the mistake is always one range resolution.**

## 5. The Separator

**Observation 5.1**

**the separator is whether the velocity has been mistaken by one wrap.Without the mistake the corrected range is $5.000014$ m; with it, $5.149901$ m, an error of $1.000033$ range resolutions——"the range shift within one chirp can be corrected with the read velocity" is not decided true or false until one says whether the velocity has been mistaken by one wrap.**

**Observation 5.2**

**beside Papers 335, 491, 649 and 266.Paper 335 showed that the round trip changes the exponent of received strength, 491 that there are two Doppler formulas, 649 that separating two periods needs a long enough record, and 266 that the premise of sampling is never met——here the round-trip $2$, the first-order Doppler formula, resolution set by observation time and sampling once per chirp are assembled in one device, and that assembly turns the wrapping of velocity into one cell of range.**

**Observation 5.3**

**what is not said.Circuits, noise, extended targets, acceleration, separation of several targets, procedures for resolving the wrap, and the numbers of actual products are not treated——what is shown is that range and velocity mix within one chirp and that a one-wrap velocity mistake becomes exactly one cell of range.**

## 6. Roll Call and Closing

1. **range resolution $\mathbf{0.149896}$ m** (Computation 2.1): **set by bandwidth alone.**
2. **velocity resolution $\mathbf{0.304173}$ m/s** (Computation 2.1): **set by observation time.**
3. **limit / resolution $\mathbf{64.000000}$** (Observation 2.3): **half the number of chirps.**
4. **$\mathbf{0.003850}$ m per m/s** (Observation 3.2): **velocity turns into range.**
5. **$\mathbf{5.038500}$ m at 10 m/s** (Computation 3.1): **appears farther.**
6. **corrected to $\mathbf{5.000014}$ m** (Observation 3.3): **back.**
7. **wrapped velocity $\mathbf{-8.935373}$ m/s** (Computation 4.1): **30 m/s beyond the limit.**
8. **corrected range $\mathbf{5.149901}$ m** (Computation 4.1): **not back.**
9. **$\mathbf{1.000033}$ resolutions** (Observation 4.2): **the core; exactly one cell.**
10. **mistaken by one wrap or not** (Observation 5.1): **the separator; Papers 335, 491, 649, 266.**

**Closing**: **A radar's range resolution is set by bandwidth and its velocity resolution by observation time, and mistaking the velocity by one wrap moves the range by exactly one cell.With 77 GHz, 1 GHz bandwidth, 50-microsecond chirps and 128 chirps, the range resolution is 0.149896 m, the velocity resolution 0.304173 m/s and the limit $\pm 19.467043$ m/s, and the ratio of limit to resolution stays at half the number of chirps, 64.000000——limit and resolution are bought one at the cost of the other.Within one chirp, 1 m/s turns into 0.003850 m of range, but correcting with the velocity read from the phase across chirps brings the range back to 5.000014 m. Yet 30 m/s, beyond the limit, wraps to $-8.935373$ m/s, and the range corrected with it, 5.149901 m, is off by 1.000033 range resolutions——because the range that one wrap of velocity, $\lambda/(2T)$, turns into is, by formula, $c/2B$ itself.The separator is whether the velocity has been mistaken by one wrap.Placed among the earlier papers——the round trip of Paper 335, the Doppler formulas of Paper 491, the record length of Paper 649 and the aliasing of sampling of Paper 266 were assembled in one device and counted.To be honest——neither circuits, noise nor extended targets are treated, and no method for resolving the wrap is stated.**

## References


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