When Sucrose Is Inverted, Sugar Content by Mass Hardly Changes, yet Freezing-Point Depression More Than Doubles

— when sucrose splits entirely into glucose and fructose, the sugar content by mass rises only **1.052641** times while the molality, and with it the dilute-solution freezing-point depression, rises **2.026665** times — the separator is whether one counts by mass or by number of particles — [Paper 803]

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Abstract

Sugar content is the mass fraction of dissolved sugar, while freezing-point depression is set by the number of dissolved particles.For sucrose inversion, in which sucrose splits into glucose and fructose, it is counted with the dilute-solution and ideal-solution formulas how far these two ways of counting return different answers.No new theorem or law is claimed. First, inversion hardly changes the sugar content by mass——from 20 to 21.052816, 1.052641 times (Section 2). **Second, and this is the core. Freezing-point depression more than doubles——molality goes from 0.730353 to 1.480182 mol/kg, 2.026665 times, and the dilute-solution depression from 1.358457 K to 2.753139 K** (Section 2). **Third, the excess over double is the water taken up——the product of 2 for the particles and 1.013333 for the loss of solvent water** (Section 3). **Fourth, compared as fractional changes, the depression moves 19.503226 times as much as the sugar content——the slope of depression per fraction inverted rises slightly from 1.380873 to 1.394681** (Section 3). **Fifth, changing the formula keeps it above double——2.003267 times with the ideal-solution formula, and 1.066679 times even against glucose dissolved from the start** (Section 4). **Sixth, the separator is whether one counts by mass or by number of particles——"the same sugar content freezes the same way" is not decided true or false until one says whether counting is by mass or by number of particles** (Section 5).
Keywords: freezing-point depression, sucrose inversion, sucrose, molality, colligative property, sugar content.

Scope of this paper (scope note): **No new theorem or law is claimed——that sucrose takes up water and splits into glucose and fructose (inversion), that freezing-point depression is a colligative property set by the number of dissolved particles (Raoult 1882), and the dilute-solution and ideal-solution formulas are all known (the textbook by Atkins and de Paula).High concentrations are not treated——only the dilute-solution and ideal-solution formulas are used, and departures in concentrated solutions (activity coefficients) are left out (the same line as Paper 333).Refractometer readings are not treated——sugar content is counted as the mass fraction dissolved, and how refractometer scales differ between sugars is not the subject.Reaction rates are not treated——the rate of inversion by acid or enzyme and the conditions setting intermediate fractions are not entered. The fraction inverted is a given value.The numbers are examples——20 g of sucrose and 80 g of water, molar masses 342.30, 180.16 and 18.015, a freezing-point constant of 1.86 K kg/mol and a heat of fusion of 6009.5 J/mol are values for comparison.No food is designed.Relation to earlier papers**: **Paper 333 showed that the constancy of the ratio of osmotic pressure to freezing-point depression follows from an assumption, not a measurement, counting the particles added by dissociation with the van ’t Hoff factor——there, dissociation added particles. Here a sugar that does not dissociate doubles its particles by reaction.Paper 718 showed that water taken up in saccharification increases mass and pushes the yield above one——here too the water taken up in inversion makes the sugar mass 1.052644 times, but for freezing-point depression that water acts not on the side added to the dissolved mass but on the side subtracted from the solvent water.**What is added is **showing that inversion makes the sugar content by mass 1.052641 times and the depression 2.026665 times**, **splitting 2.026665 into 2 for the particles and 1.013333 for the loss of water**, **deriving 19.503226 as the ratio of fractional changes and confirming 2.003267 with the ideal-solution formula**, and **placing the separator on whether one counts by mass or by number of particles**.

1. Notation and the Question

Setting: **sucrose takes up one water molecule and splits into two, glucose and fructose (sucrose inversion).**

$$\mathrm{C_{12}H_{22}O_{11} + H_2O \longrightarrow C_6H_{12}O_6 + C_6H_{12}O_6}$$

Let w_{s} be the mass of dissolved sugar, w_{w} the mass of solvent water and n the amount of dissolved particles, and count the sugar content by mass Bx, the molality b and the freezing-point depression as follows.

$$\mathrm{Bx} = 100\,\frac{w_{\mathrm{s}}}{w_{\mathrm{s}} + w_{\mathrm{w}}}, \qquad b = \frac{n}{w_{\mathrm{w}}}, \qquad \Delta T_{\mathrm{dilute}} = K_f\, b, \qquad \ln x_{\mathrm{w}} = -\frac{\Delta H}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

The last is the ideal-solution formula, with x_{w} the mole fraction of water. Start from 20 g of sucrose and 80 g of water.

Question: **under inversion, by what factor does a quantity counted by mass change, and one counted by number of particles?**

2. Complete Inversion

Computation 2.1: **before inversion and after complete inversion were set side by side.**

quantity	before	after complete inversion	factor
sugar content by mass	**20.000000**	**21.052816**	**1.052641**
solvent water	**80.000000** g	**78.947415** g	
molality	**0.730353** mol/kg	**1.480182** mol/kg	**2.026665**
depression (dilute-solution formula)	**1.358457** K	**2.753139** K	**2.026665**
depression (ideal-solution formula)	**1.342715** K	**2.689818** K	**2.003267**

> **Observation 2.2**
>
> **inversion hardly changes the sugar content by mass.The sugar content goes from 20.000000 to 21.052816, 1.052641 times, the increase being only the water taken up into the sugar's mass——as long as one counts by mass, inversion is almost invisible.**

> **Observation 2.3**
>
> **this is the core: freezing-point depression more than doubles.Molality becomes 2.026665 times, and the dilute-solution depression moves from 1.358457 K to 2.753139 K——counting the same solution by particles, inversion appears as a change of more than double.**

3. The Excess over Double

Computation 3.1: **the molality factor was split into the factor for the number of particles and the loss of solvent water.**

$$\frac{b_{\text{after}}}{b_{\text{before}}} = 2 \times \frac{80}{78.947415} = 2 \times 1.013333 = 2.026665$$

The increase in sugar mass is 1.052644 by counting substances and 1.052629 from the sum of the molar masses of sucrose and water, the gap being the 0.005 g/mol of rounding in molar masses.

Observation 3.2

the excess over double is the water taken up.The water taken up is added to the sugar's mass and subtracted from the solvent water. On sugar content it acts as a slight gain in mass; on freezing-point depression, as the loss of solvent water, 1.013333——the same water enters opposite sides of the two ways of counting.

Computation 3.3: the fraction inverted was moved from 0 to 1.

fraction inverted	sugar content by mass	molality	depression (dilute)	depression (ideal)
0.00	20.000000	0.730353	1.358457 K	1.342715 K
0.25	20.263205	0.915955	1.703676 K	1.679074 K
0.50	20.526409	1.102785	2.051180 K	2.015710 K
0.75	20.789613	1.290857	2.400994 K	2.352623 K
1.00	21.052816	1.480182	2.753139 K	2.689818 K

Observation 3.4

compared as fractional changes, the depression moves 19.503226 times as much as the sugar content.Dividing the fractional increase in depression, 1.026665, by that in sugar content, 0.052641, gives 19.503226. The slopes of depression up to fractions 0.25, 0.5, 0.75 and 1 are 1.380873, 1.385446, 1.390048 and 1.394681, rising slightly——as inversion proceeds the solvent water shrinks, so each step of inversion moves the depression slightly more.

4. Changing the Formula, Changing the Sugar

Computation 4.1: the freezing-point constant 1.86 K kg/mol agrees with $RT_0^2 M_\mathrm{w}/\Delta H = 1.859656$ K kg/mol from the heat of fusion 6009.5 J/mol. With the ideal-solution formula, the ratio of depressions before and after inversion is 2.003267.

Computation 4.2: dissolving 20 g of glucose in 80 g of water from the start instead of sucrose gives a molality of 1.387655 mol/kg, 1.899978 times that of sucrose (the same as the ratio of molar masses). After complete inversion the molality is a further 1.066679 times that.

Observation 4.3

changing the formula keeps it above double.2.026665 times with the dilute-solution formula and 2.003267 times with the ideal-solution formula, both above 2——being above double is not a product of the dilute-solution approximation.

Observation 4.4

inversion gives more particles than dissolving the same mass of monosaccharide from the start.1.066679 is the product of the gain in sugar mass, 1.052644, and the loss of solvent water, 1.013333——inversion takes up water, making the sugar heavier and at the same time reducing the solvent water.

5. The Separator

Observation 5.1

the separator is whether one counts by mass or by number of particles.Sugar content counted by mass becomes 1.052641 times, freezing-point depression counted by particles 2.026665 times, and their fractional changes differ by a factor of 19.503226——“the same sugar content freezes the same way” is not decided true or false until one says whether counting is by mass or by number of particles.

Observation 5.2

beside Papers 333 and 718.Paper 333 counted particles added by dissociation with a factor, and Paper 718 counted mass exceeding one through water taken up——here a sugar that does not dissociate adds particles by reaction, and the same water taken up is added to the mass and subtracted from the solvent.

Observation 5.3

what is not said.Departures in concentrated solutions, activity coefficients, refractometer readings, rates of inversion and food design are not treated——what is shown is that the same inversion acts on mass and on number of particles by different factors.

6. Roll Call and Closing

1. **sugar content by mass 21.052816** (Computation 2.1): **hardly changed.**
2. **factor in sugar content 1.052641** (Observation 2.2): **the water taken up.**
3. **factor in molality 2.026665** (Observation 2.3): **the core.**
4. **depression 2.753139 K** (Computation 2.1): **dilute-solution formula.**
5. **loss of water 1.013333** (Observation 3.2): **the excess over double.**
6. **ratio of fractional changes 19.503226** (Observation 3.4): **the ways of counting part widely.**
7. **2.003267 times with the ideal-solution formula** (Observation 4.3): **not an artefact of approximation.**
8. **freezing-point constant 1.859656** (Computation 4.1): **from the heat of fusion.**
9. **1.066679 times against glucose from the start** (Observation 4.4): **heavier, and less water.**
10. **mass or number of particles** (Observation 5.1): **the separator; Papers 333, 718.**

Closing: **When sucrose is inverted, sugar content by mass hardly changes, yet freezing-point depression more than doubles.When a solution of 20 g of sucrose in 80 g of water splits entirely into glucose and fructose, the sugar content by mass rises only to 21.052816 (1.052641 times)——as long as one counts by mass, inversion is almost invisible.Yet molality becomes 2.026665 times and the dilute-solution depression moves from 1.358457 K to 2.753139 K, that 2.026665 being the product of 2 for the particles and 1.013333 for the loss of solvent water, and the ratio is still 2.003267 with the ideal-solution formula——the same water taken up is added to the mass and subtracted from the solvent.The separator is whether one counts by mass or by number of particles.Placed among the earlier papers——Paper 333 counted particles added by dissociation, and Paper 718 mass exceeding one through water taken up. This counts a sugar that does not dissociate adding particles by reaction.To be honest——neither departures in concentrated solutions, refractometer readings, nor rates of inversion are treated.**

References

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