Clock Longitude Errors Grow with the Days; One Lunar Arcminute Is as Many Longitude Arcminutes as the Sidereal Month Has Days

— misreading the Moon--star distance by **1** arcminute shifts the longitude by **27.321661** arcminutes, and a clock with a daily-rate error of **1** second draws level with it after **109.286644** days — the separator is whether the clock's error accumulates with the days or the sky's clock does not — [Paper 805]

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— From thinking to syncing. —

Abstract

Knowing longitude at sea needs the difference between the time at a reference place and the ship's local time.It is counted how the error differs when that reference time is read from a carried clock or from the sky's clock, the angular distance between the Moon and a star.No new theorem or law is claimed. First, one arcminute of the Moon is 109.286644 seconds of time——the Moon moves only 32.940896 arcminutes per hour against the stars (Section 2). **Second, in longitude it is 27.321661 arcminutes, the same number as the days of the sidereal month——the time shift is four times the sidereal-month days in seconds and the longitude shift is that many arcminutes, agreeing by formula** (Section 2). **Third, and this is the core. A clock's error accumulates with the days; the Moon's does not——a clock with a 1-second daily-rate error draws level with the Moon's 1 arcminute after 109.286644 days, and one with 5 seconds after 21.857329 days** (Section 3). **Fourth, the Moon's clock is not evenly graduated——with eccentricity 0.0549, the longitude shift per arcminute moves between 24.440951 and 30.449850 arcminutes** (Section 2). **Fifth, the same angular error becomes a shorter distance at higher latitude——the Moon's 1 arcminute is 50.599716 km at the equator and 32.524871 km at latitude 50 degrees** (Section 4). **Sixth, the separator is whether the clock's error accumulates with the days or the sky's clock does not——"the clock is more accurate" is not decided true or false until one says how many days the voyage lasts** (Section 5). **Keywords**: longitude, lunar distances, sidereal month, daily-rate error, carried clock, eccentricity.

Scope of this paper (scope note): **No new theorem or law is claimed——finding longitude from a time difference (one hour of time is 15 degrees of longitude), reading the reference time from the angular distance between the Moon and a star (the method of lunar distances), and the length of the sidereal month are all known.Navigation is not taught——how lunar distances are actually measured, corrections for parallax and refraction, use of tables, and observing procedures are not entered.History is not treated——the story of the longitude problem, the making of clocks, prizes, and the performance of actual clocks are not the subject.The Moon's motion is counted by its mean——the mean angular speed over the sidereal month is used; the wavering due to eccentricity is shown only as a range, and perturbations such as the Sun's are left out.The ship's local time is taken as correctly found——the error in finding local time applies equally to both methods, and was removed for the comparison.The clock's error is set as one number——the daily-rate error is a constant in seconds per day, and its fluctuation is not treated (the distinction between bias and fluctuation is left to Paper 255).Relation to earlier papers**: **Paper 596 showed that the earliest sunset comes half a month before the winter solstice and that the gap between sundial and clock is the sum of two causes, and its scope note stated that the history of calendars and clocks was not treated——this paper stands beside that seat: here too clock and sky time are compared, but what is counted is not the cause of the gap but how the error of reading longitude accumulates.Paper 340 showed that the shorter path is not the one a navigator steers, and its scope note stated that navigation was not taught——this paper also stands beside that seat and teaches no navigation.Paper 406 showed, using the sidereal month, that seeing the same face is the evidence that the Moon rotates——here the same number of sidereal-month days becomes the spread of the longitude error itself.Paper 535 showed that what fixes how much of the Moon is visible is which side two normals lie on, using the Moon's orbital eccentricity 0.0549——here the same eccentricity makes the pace of the Moon's clock waver.Paper 255 showed that "accuracy" is two things and calibration removes only one——this paper sets the daily-rate error as one number for what calibration leaves, without entering its content.**What is added is **showing by formula and bisection that the longitude shift per arcminute of the Moon equals the number of sidereal-month days**, **deriving, for each daily-rate error, the days after which a clock draws level with the Moon**, **giving the wavering of the Moon's clock by eccentricity as a range**, and **placing the separator on a clock whose error accumulates versus a sky's clock whose error does not**.

1. Notation and the Question

Setting: **the longitude difference $\Delta\lambda$ is found from the difference Δt between the reference time and the ship's local time. One second of time is 15 arcseconds of longitude. The reference time is read in two ways.**

$$\Delta\lambda = 15\,\Delta t, \qquad \omega_{\mathrm{M}} = \frac{360^\circ}{P \times 24\ \text{hours}}, \qquad \delta t_{\text{Moon}} = \frac{\delta d}{\omega_{\mathrm{M}}}, \qquad \delta t_{\text{clock}} = r\,D$$

P is the number of days of the sidereal month, ω_{M} the Moon's angular speed against the stars, δd the misreading of the Moon--star distance, r the clock's daily-rate error (seconds per day), and D the days at sea.

Question: **how does the error of reading longitude differ between clock and Moon, and where do they change places?**

2. The Moon’s Clock

Computation 2.1: **with a sidereal month of $P = 27.321661$ days, the shifts from misreading the Moon’s angular distance by 1 arcminute were derived.**

quantity	value
the Moon’s angular speed against the stars	**32.940896** arcmin/hour (**0.549015** deg/hour)
time from a **1**-arcminute misreading	**109.286644** s (bisection also **109.286644** s)
longitude from a **1**-arcminute misreading	**27.321661** arcmin (**0.455361** deg)

> **Observation 2.2**
>
> **the longitude shift equals the number of sidereal-month days.By the following formula, the time shift is $4P$ seconds and the longitude shift P arcminutes, differing from the computed value by -3.553×10^{-15}——that the Moon is a slow clock spreads the longitude error by exactly the number of sidereal-month days.**

$$\delta d = 1' \;\Rightarrow\; \delta t_{\text{Moon}} = \frac{60''}{15''/P \text{ per second}} = 4P \text{ s}, \qquad \delta\lambda_{\text{Moon}} = 15 \times 4P \text{ arcsec} = P \text{ arcmin}$$

Computation 2.3: **with the Moon’s orbital eccentricity 0.0549, its angular speed wavers between 0.897268 and 1.117864 times the mean (ratio 1.245854). The longitude shift per arcminute ranges from 24.440951 to 30.449850 arcminutes.**

> **Observation 2.4**
>
> **the Moon’s clock is not evenly graduated.The same 1-arcminute misreading becomes 24.440951 or 30.449850 arcminutes depending on where the Moon is in its orbit——the sky’s clock has graduations that stretch and shrink within each month.**

3. The Carried Clock

Computation 3.1: **for each daily-rate error r, the days after which the clock’s shift equals the Moon’s 1 arcminute, and the longitude shift on a 42-day voyage, were derived.**

daily-rate error	days to draw level with 1 arcmin	longitude shift after 42 days
0.5 s/day	218.573288 days	5.250000 arcmin
1 s/day	109.286644 days	10.500000 arcmin
2 s/day	54.643322 days	21.000000 arcmin
5 s/day	21.857329 days	52.500000 arcmin

> **Observation 3.2**
>
> **this is the core: a clock's error accumulates with the days; the Moon's does not.The clock's shift rD is proportional to the days at sea, while the Moon's is set only by the misread angle, independent of the days. They draw level after the following number of days: 109.286644 for a 1-second daily-rate error, 21.857329 for 5 seconds——on short voyages the clock wins; on long ones the Moon, read to one arcminute, wins.**

$$D^{*} = \frac{4P}{r} \text{ days} \qquad (r \text{ in seconds per day})$$

> **Observation 3.3**
>
> **the days for a 1-second clock to draw level equal the seconds of time in the Moon's 1 arcminute.Both are 109.286644——accumulating 1 second per day over $4P$ days reaches the $4P$ seconds of the Moon's 1 arcminute.**

4. Into Distance

Computation 4.1: **longitude shifts were converted into east--west distance (1 arcminute is 1.852 km at the equator, times $\cos\varphi$ at latitude φ).**

latitude	the Moon's 1-arcmin misreading	the clock's 1 second
0 deg	50.599716 km	0.463000 km
30 deg	43.820640 km	0.400970 km
50 deg	32.524871 km	0.297611 km

Observation 4.2

the same angular error becomes a shorter distance at higher latitude.Because an arcminute of longitude shrinks by $\cos\varphi$, the Moon's misreading and the clock's second shrink in the same proportion——which method wins does not change with latitude, only with the days at sea.

5. The Separator

Observation 5.1

the separator is whether the clock's error accumulates with the days or the sky's clock does not.The Moon's 1 arcminute is 109.286644 seconds of time and 27.321661 arcminutes of longitude, independent of the days, while a clock accumulates its daily-rate error times the days——"the clock is more accurate" is not decided true or false until one says how many days the voyage lasts.

Observation 5.2

beside Papers 596, 340, 406 and 535.Paper 596 counted the causes of the gap between clock and sky, 340 the choice of route, and 406 and 535 how the Moon is seen, using the sidereal month and the eccentricity——here the same sidereal month and eccentricity become the graduations of the sky's clock for reading longitude, and it was counted that its error accumulates differently from a carried clock.

Observation 5.3

what is not said.How lunar distances are measured and corrected, use of tables, the history of the longitude problem, the performance of actual clocks, fluctuation of the daily rate, and solar perturbations are not treated——what is shown is the difference in how the two clocks' errors accumulate.

6. Roll Call and Closing

1. **the Moon's angular speed 32.940896 arcmin/hour** (Computation 2.1): **a slow clock.**
2. **1 arcminute is 109.286644 s of time** (Computation 2.1): **four times the sidereal month in seconds.**
3. **longitude 27.321661 arcmin** (Observation 2.2): **the sidereal-month days.**

4. **from 24.440951 arcmin by eccentricity** (Computation 2.3): **graduations shrink.**
5. **to 30.449850 arcmin** (Computation 2.3): **graduations stretch.**
6. **109.286644 days for a 1-second clock** (Observation 3.2): **the core.**
7. **21.857329 days at 5 seconds** (Computation 3.1): **the pace of accumulation.**
8. **50.599716 km at the equator** (Computation 4.1): **the Moon's arcminute in distance.**
9. **32.524871 km at latitude 50 deg** (Computation 4.1): **shrinks in proportion.**
10. **an accumulating clock or a non-accumulating sky clock** (Observation 5.1): **the separator; Papers 596, 340, 406, 535.**

Closing: **The error in reading longitude by clock accumulates with the days at sea and that by the Moon does not, but one arcminute of the Moon becomes as many arcminutes of longitude as the sidereal month has days.The Moon moves only 32.940896 arcminutes per hour against the stars, so a 1-arcminute misreading of its distance is 109.286644 seconds of time and 27.321661 arcminutes of longitude, agreeing by formula with the days of the sidereal month——the Moon is a slow clock.A clock's error accumulates as daily-rate error times days, so a clock with a 1-second error draws level with the Moon's 1 arcminute after 109.286644 days and one with 5 seconds after 21.857329 days, beyond which the Moon wins. The Moon's graduations waver between 24.440951 and 30.449850 arcminutes with eccentricity 0.0549——it does not accumulate, but its graduations waver.The separator is whether the clock's error accumulates with the days or the sky's clock does not.Placed among the earlier papers——standing beside the seats of Papers 596 and 340, the sidereal month of Paper 406 and the eccentricity of Paper 535 were counted as the graduations of a clock for reading longitude.To be honest——neither the measurement and correction of lunar distances nor history is treated, and the clock's error was set as one number.**

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