The Same Formulas Split Into Four Kinds by Truth Tables and Into Six by Widening Worlds

— Of **3303** one-variable formulas, truth tables give only the **4** kinds always-false, P, $\neg P$, always-true, while **828** Kripke models give **6** — the separator is whether one splits by truth tables or by widening worlds — [Paper 808]

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Abstract

Reading “holds” as decided by a truth table, or as surviving however the worlds widen, splits the same collection of formulas in two different ways.All one-variable formulas up to depth 2 were built, and the kinds were counted under both splittings.No new theorem or law is claimed. First, truth tables give only 4 kinds, however far the depth is increased——both the 33 formulas of depth 1 and the 3303 of depth 2 give always-false, P, $\neg P$, always-true (Section 2). **Second, and this is the core. Splitting by widening worlds increases the kinds——with the 828 Kripke models on up to 3 worlds, depth 1 gives 4 kinds and depth 2 gives 6 (1.500000 times)** (Section 3). **Third, three fail and one survives——excluded middle, double-negation elimination and Peirce’s law fail, while double-negation introduction holds in every model; the smallest counterexample has 2 worlds** (Section 4). **Fourth, the failures are a small part of the whole——of 1667 classical tautologies, 1635 (0.980804) still hold once the worlds widen, and only 32 (0.019196) fail** (Section 4). **Fifth, adding a double negation lets every failed formula through——in the form $\neg\neg\varphi$ all 1667 hold (1.000000)** (Section 5). **Sixth, the separator is whether one splits by truth tables or by widening worlds ——“are these two formulas the same?” has no answer until one says under which splitting the question is asked** (Section 6). **Keywords**: intuitionistic logic, Kripke model, excluded middle, double negation, Glivenko’s theorem, tautology.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——Kripke semantics, the failure of excluded middle and double-negation elimination in intuitionistic logic, Glivenko's theorem, and the infinitely many kinds of one-variable formula (Rieger--Nishimura) are all known.No theorem is proved——Glivenko's theorem and Kripke completeness are only cited.The count is finite——it covers one variable, formulas up to depth 2 (3303 of them), and models on up to 3 worlds (828 of them); that the kinds increase without bound is not checked here.No position is argued——whether constructivism is right, and how the foundations should be laid, are not the subject.Only one variable——how the kinds grow with two or more variables is not entered.Relation to earlier papers**: **Paper 179 showed that "cannot be constructed" has distinct roots——this paper stands beside it: there the distinct roots of non-constructibility, here the distinct roots of "holds".Paper 155 built spaces with no points at all——excluded middle appeared there too, but the stage was space; here the formulas themselves are counted.Paper 300 showed that whether two things share a root is decidable——the two splittings here are distinct roots, and no agreement of numbers is written up as sameness.**What is added is **counting that the same collection splits into 4 and 6 kinds**, **counting that only 32 classical tautologies (0.019196) fail**, **confirming that all 1667 pass once a double negation is added**, and **placing the separator on truth tables versus widening worlds**.

1. Notation and the Question

Setting: **formulas are built from one variable P and the constants $\top$, $\bot$ with $\neg$, $\wedge$, $\vee$, $\to$. There are 33 of them at depth 1 and 3303 at depth 2.**

There are two ways to split them.One is the truth table: put 0 and 1 for P and count formulas with the same values as one.The other is the Kripke model: take a collection of worlds with a relation "one can move on from here", and count formulas as one when the same worlds force them.

$$w \Vdash \neg A \iff \forall v \geq w :\ v \nVdash A, \qquad w \Vdash A \to B \iff \forall v \geq w :\ (v \Vdash A \Rightarrow v \Vdash B)$$

Taking every partial order on up to 3 worlds together with every upward-closed valuation of P gives 828 models.

Question: **into how many kinds does the same collection of formulas split under the two ways?**

2. Splitting by Truth Tables

Computation 2.1: **the kinds under the truth table were counted at two depths.**

depth	formulas	kinds by truth table
1	33	4
2	3303	4

> **Observation 2.2**
>
> truth tables give only 4 kinds, however far the depth is increased.The four are always-false, P, $\neg P$, always-true——a function of one variable is a map from $\{0,1\}$ to $\{0,1\}$, and there are only $2^2 = 4$ of those.

3. Splitting by Widening Worlds

Computation 3.1: the same formulas were split by the 828 Kripke models on up to 3 worlds.

depth	formulas	truth table	Kripke models
1	33	4	4
2	3303	4	6

> **Observation 3.2**
>
> this is the core. Splitting by widening worlds increases the kinds.At depth 1 both give 4 and nothing separates them; at depth 2 the truth table stays at 4 while Kripke gives 6 (1.500000 times)——a truth table asks only “is it true now”, while a Kripke model asks “is it true however far one goes on”.

4. What Fails and What Survives

Computation 4.1: four formulas were checked in every model.

formula	holds in every model
excluded middle $P \vee \neg P$	no
double-negation elimination $\neg\neg P \to P$	no
Peirce's law $((P \to \bot) \to P) \to P$	no
double-negation introduction $P \to \neg\neg P$	yes

> **Observation 4.2**
>
> **the smallest counterexample has 2 worlds.From the lower world one can move to the upper one, and P is true only above. At the lower world P is not true, and since P becomes true above, $\neg P$ is not true either——the moment "not true now" parts from "never becomes true", excluded middle falls.**

Computation 4.3: **among the formulas up to depth 2, those with value 1 in every row of the truth table (the classical tautologies) were counted, and then those holding in every model.**

counted	number	share
classical tautologies	**1667**	**1.000000**
still holding once worlds widen	**1635**	**0.980804**
failing	**32**	**0.019196**

> **Observation 4.4**
>
> **the failures are a small part of the whole.Of the 1667 classical tautologies only 32 (0.019196) fail, and 1635 (0.980804) pass unchanged——the two readings agree on almost every formula and part only on a few at the boundary.**

5. Adding a Double Negation

Computation 5.1: **for each of the 1667 classical tautologies, $\neg\neg\varphi$ was checked in every model.**

Observation 5.2

adding a double negation lets every failed formula through.All 1667 satisfied $\neg\neg\varphi$ (1.000000)——this is what Glivenko's theorem says: what failed was the strength of "holds", not the content.

6. The Separator

Observation 6.1

the separator is whether one splits by truth tables or by widening worlds.The same 3303 formulas fall into 4 kinds one way and 6 the other——"are these two formulas the same?" has no answer until one says under which splitting the question is asked.

Observation 6.2

placed beside Papers 179, 155 and 300.Paper 179 showed distinct roots of "cannot be constructed", Paper 155 built spaces with no points, and Paper 300 showed that sharing a root is decidable——here the distinct roots of "holds" are separated by one quantity, the number of kinds.

Observation 6.3

what is not said.Which logic is right, whether constructivism should be adopted, and how the kinds grow with more variables are not treated——what is shown is that the same collection splits into 4 and 6.

7. Census and Closing

1. **3303 formulas, 828 models** (Computations 2.1, 3.1): **the range counted.**
2. **4 kinds by truth table** (Observation 2.2): **depth does not change it.**
3. **6 kinds by Kripke** (Observation 3.2): **the core; 1.500000 times.**
4. **excluded middle fails** (Computation 4.1): **counterexample on 2 worlds.**
5. **double-negation elimination fails** (Computation 4.1): **so does Peirce's law.**
6. **double-negation introduction survives** (Computation 4.1): **direction matters.**

7. **1667 classical tautologies** (Computation 4.3): **the denominator.**
8. **1635 pass (0.980804)** (Observation 4.4): **almost all agree.**
9. **32 fail (0.019196)** (Observation 4.4): **a few at the boundary.**
10. **all 1667 with a double negation** (Observation 5.2): **Glivenko; 1.000000.**

Closing: **The same formulas split into four kinds by truth tables and into six by widening worlds.Built up to depth 2, one-variable formulas number 3303, and the truth table gives only always-false, P, $\neg P$, always-true however far the depth goes——because there are only 4 functions of one variable.The 828 Kripke models on up to 3 worlds give 6 kinds, with excluded middle, double-negation elimination and Peirce's law failing and only double-negation introduction surviving——because "not true now" parts from "never becomes true".Only 32 of the 1667 classical tautologies (0.019196) fail, and with a double negation all 1667 pass.The separator is whether one splits by truth tables or by widening worlds.Placed among the earlier papers——standing beside Paper 179's distinct roots of "cannot be constructed", counting formulas rather than Paper 155's spaces, and keeping Paper 300's discipline of distinct roots.To be honest——these are counts over a finite range, one variable, depth 2, at most 3 worlds, and the unbounded growth of the kinds is not checked here.**

References

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