Beyond “There Must Be One,” It Is Not Search but the Distribution of Degrees That Decides

— Building all **32768** two-colourings of the complete graph on **6** points gives a minimum of **2** monochromatic triangles, with neither **0** nor **1** possible — the separator is whether one searches the colourings or the degree distribution decides — [Paper 809]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— From thinking to syncing. —

Abstract

Ramsey’s theorem says that among 6 people there must be three who all know one another or three who are all strangers.This paper strengthens “there must be one” to “there must be at least how many” and counts what fixes that number.No new theorem or law is claimed. First, on 5 points the count can be 0——12 of the 1024 colourings (0.011719) have no monochromatic triangle at all, which is the content of $R(3,3) > 5$ (Section 2). **Second, on 6 points the minimum is 2, and no colouring gives 1——over all 32768 colourings the minimum is 2, reached by 1760 of them (0.053711), with mean 5.000000** (Section 2). **Third, and this is the core. The count is fixed by the distribution of red degrees alone——Goodman’s identity matched in all 33792 colourings on 5 and 6 points** (Section 3). **Fourth, so the lower bound comes out without looking at a single colouring——using only the degree distribution and the handshake condition (the degrees sum to an even number), it is 2 for 6 points, 4 for 7, and 40 for 12 (20.000000 times the value at 6)** (Section 3). **Fifth, the search side explodes over the same range——the number of colourings grows from 32768 at 6 points to 7.378698×10^{19} at 12** (Section 4). **Sixth, the separator is whether one searches the colourings or the degree distribution decides——“how many are there” looks like a question for search, yet the lower-bound side needs none** (Section 4). **Keywords**: Ramsey’s theorem, monochromatic triangle, Goodman’s identity, degree distribution, exhaustive enumeration, lower bound.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——Ramsey's theorem, $R(3,3)=6$, and Goodman's identity and bound for the number of monochromatic triangles are all known.The identity is not proved——it is only checked against the exhaustive enumeration on 5 and 6 points.Only two colours and triangles——three or more colours, monochromatic subgraphs on four or more points, and asymmetric Ramsey numbers are not the subject.Nothing above 6 points is enumerated——for 7 points and above only the bound from the degree distribution is written.No Ramsey number is claimed——neither the values nor the ranges of $R(4,4)$ or $R(5,5)$ are entered.Relation to earlier papers**: **Paper 302 showed that "it must appear" does not tell you when, and its scope note stated "no Ramsey number is computed—the value of $R(5,5)$ is not sought; only the effort required to seek it is counted"——this paper stands beside that seat, deriving not the value but the lower bound on the count, without any search.Paper 300 showed that whether two things share a root is decidable——the bound and the search effort here are distinct roots that merely share a stage.**What is added is **enumerating all colourings on 5 and 6 points for the minimum, the mean and the number of colourings achieving it**, **confirming Goodman's identity in all 33792 colourings**, **listing the bound up to 12 points and measuring the 20.000000-fold rise**, and **placing the separator on search versus the degree distribution**.

1. Notation and the Question

Setting: **the edges of the complete graph on n points are coloured in two colours, red and blue. A triple whose three edges share a colour is a monochromatic triangle.Write d_v for the number of red edges at a vertex v.**

The first case of Ramsey's theorem is $R(3,3)=6$: on 6 points a monochromatic triangle must occur, while on 5 points one can colour so that none does.

Question: **when "there must be one" is strengthened to "there must be at least how many", what fixes that number?**

2. Building Every Colouring

Computation 2.1: **every colouring on 5 points (1024) and on 6 points (32768) was built and its monochromatic triangles counted.**

points	colourings	minimum	colourings reaching it	mean
5	1024	0	12 (0.011719)	2.500000
6	32768	2	1760 (0.053711)	5.000000

Observation 2.2

on 6 points the minimum is neither 0 nor 1 but 2.On 5 points 12 colourings reach 0 (the pentagon with its diagonals in the other colour), while on 6 points every colouring gives at least 2——the phrase “there must be one” is weaker than what is actually guaranteed.

3. Fixed by the Degrees Alone

Computation 3.1: **Goodman’s identity was checked against every colouring built.**

$$\#\{\text{monochromatic triangles}\} = \binom{n}{3} - \frac{1}{2}\sum_{v} d_v\,(n-1-d_v)$$

In all 33792 colourings on 5 and 6 points the enumerated count and the identity agreed.

Observation 3.2

this is the core. The count is fixed by the distribution of red degrees alone.Only the number of red edges at each vertex enters the right-hand side; which particular edge carries which colour does not——colourings with the same degree distribution have the same number of monochromatic triangles, however differently they are arranged.

Computation 3.3: **the degree distribution maximising $\sum_v d_v(n-1-d_v)$ was chosen subject to the handshake condition that the degrees sum to an even number, giving the bound.**

points	5	6	7	8	9	10	11	12
bound	0	2	4	8	12	20	28	40

Observation 3.4

the bound comes out without looking at a single colouring.The bound of 40 at 12 points is 20.000000 times the 2 at 6 points, and all it took was the degree distribution and the handshake condition——when the number of points is odd, not every vertex can take $(n-1)/2$, and the bound rises by exactly that much.

4. The Search Side, and the Separator

Observation 4.1

the search side explodes over the same range.The number of colourings is 32768 at 6 points and 7.378698×10^{19} at 12——while the bound needs only the handshake condition, finding a colouring that attains the minimum needs enumeration.

Observation 4.2

the separator is whether one searches the colourings or the degree distribution decides.One question, “how many monochromatic triangles are there”, is answered by a closed formula if only the bound is wanted and by enumeration if an attaining colouring is wanted——a side that needs search and a side that does not live inside the same question.

Observation 4.3

placed beside Papers 302 and 300.Paper 302 counted the effort of brute-forcing $R(5,5)$ and stated that no value was sought——here it is not a value but a lower bound on a count, obtained without search.Paper 300 showed that sharing a root is decidable——the bound and the search effort are distinct roots sharing a stage.

Observation 4.4

what is not said.Three or more colours, larger monochromatic subgraphs, the values of $R(4,4)$ and $R(5,5)$, and enumeration beyond 6 points are not treated——what is shown is that the lower bound on the count is fixed by the degree distribution alone.

5. Census and Closing

1. **12 colourings reach 0 on 5 points (0.011719)** (Computation 2.1): **the content of $R(3,3) > 5$.**
2. **minimum 2 on 6 points** (Observation 2.2): **neither 0 nor 1.**
3. **reached by 1760 (0.053711)** (Computation 2.1): **a narrow set.**

4. **mean 5.000000 on 6 points** (Computation 2.1): **above the minimum.**
5. **identity matches in 33792** (Computation 3.1): **the core.**
6. **only degrees enter** (Observation 3.2): **arrangement does not.**
7. **bound 2 at 6, 4 at 7** (Computation 3.3): **no search.**
8. **40 at 12 (20.000000 times)** (Observation 3.4): **the handshake condition bites.**
9. **7.378698×10^{19} colourings** (Observation 4.1): **the search side explodes.**
10. **search or degree distribution** (Observation 4.2): **the separator; Papers 302, 300.**

Closing: **Beyond "there must be one," it is not search but the distribution of degrees that decides.All 32768 colourings on 6 points give a minimum of 2 monochromatic triangles, with neither 0 nor 1 possible, while on 5 points 12 of 1024 (0.011719) reach 0——that is exactly the boundary $R(3,3) = 6$.Goodman's identity fixes the count from the red degrees alone (matching in all 33792 colourings), so the lower bound comes out without looking at a single colouring: 2 at 6 points and 40 at 12 (20.000000 times)——because no arrangement enters the right-hand side.Over the same range the number of colourings rises to 7.378698×10^{19}, and the separator is whether one searches the colourings or the degree distribution decides.Placed among the earlier papers——standing beside Paper 302's seat for not computing Ramsey numbers, and keeping Paper 300's discipline in treating the bound and the search effort as distinct roots.To be honest——the enumeration reaches only 6 points, and for 7 and above all that is said is the bound.**

References

1. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
2. Yamagishi, Y. (2026). *"It Must Appear" Does Not Tell You When* [Paper 302]. Zenodo. 10.5281/zenodo.22171988

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)