Only Four Numbers Sit Exactly on the Equality Below a Million, While the Inequality Just Above It Takes a Quarter

— Numbers satisfying $\sigma(n) = 2n$ exactly number 4 below 10^6 (6, 28, 496, 8128; a share of 0.000004000) — the separator is whether one asks for exact equality or for exceedance — [Paper 810]

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September 2026

— *From thinking to syncing.* —

Abstract

A perfect number is defined by an equality: the sum of its divisors is exactly twice itself.This paper enumerates up to 10^6 and sets the density of the numbers sitting on that equality beside the density of those satisfying the inequality just above it.No new theorem or law is claimed. First, numbers on the equality are extremely rare——below 10^6 there are just 4, namely 6, 28, 496 and 8128, a share of 0.000004000 (Section 2). **Second, and this is the core. The inequality just above takes a quarter——abundant numbers number 247545 (0.247545) and deficient ones 752450 (0.752451), a ratio to the equality of 1 to 61886.2** (Section 2). **Third, the neighbourhood of the boundary is crowded too——numbers with $\sigma(n)/2n$ between 0.99 and 1.01 number 34212 (0.034212), which is 8553.0 times those exactly on it** (Section 3). **Fourth, among odd numbers only the inequality appeared——there are 1996 odd abundant numbers, the smallest 945, and not one odd perfect number below 10^6** (Section 3). **Fifth, numbers on the equality can be built rather than sought——the form $2^{p-1}(2^p - 1)$ satisfies $\sigma(N) = 2N$ for all of $p = 2, 3, 5, 7, 13, 17, 19, 31$, reaching $N = 2305843008139952128$ at $p = 31$** (Section 4). **Sixth, the separator is whether one asks for exact equality or for exceedance——on the same σ, the equality has almost no density while the inequality has a positive one** (Section 5). **Keywords**: perfect number, abundant number, deficient number, sum of divisors, Euclid--Euler classification, density.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——the definitions of perfect, abundant and deficient numbers, the Euclid--Euler classification of even perfect numbers, and the existence of a natural density for the abundant numbers (Davenport) are all known.The enumeration reaches 10^6——nothing beyond that is counted.No claim that odd perfect numbers do not exist——only that none occurred below 10^6; the question is open.Mersenne primes are not discussed——neither whether they are finite or infinite in number, nor the primality tests themselves.The density value is not claimed——0.247545 is the share below 10^6 and is not written as the natural density itself.Relation to earlier papers**: **Paper 247 showed that splitting by parity is not one phenomenon, and its scope note cited the Euclid--Euler classification of even perfect numbers as known——this paper stands beside that seat: there the distinct roots of parity splitting, here the densities of an equality and an inequality.Paper 300 showed that whether two things share a root is decidable——the rarity of the equality and the density of the inequality are two sides of one σ, and no agreement of numbers is written up as sameness.**What is added is **counting both sides up to 10^6 and measuring the ratio 1 to 61886.2**, **counting the 34212 numbers in the one-percent band around the boundary**, **confirming that only the inequality appears among odd numbers**, and **placing the separator on exact equality versus exceedance**.

1. Notation and the Question

Setting: **let $\sigma(n)$ be the sum of the divisors of n. Call n perfect when $\sigma(n) = 2n$, abundant when $\sigma(n) > 2n$, and deficient when $\sigma(n) < 2n$.**

Every σ up to 10^6 was built with a sieve and the three cases counted.

Question: **which is more numerous, the numbers sitting exactly on the equality or those satisfying the inequality just above it, and by how much?**

2. The Enumeration

Computation 2.1: **the 999999 numbers from 2 to 10^6 were split into three cases.**

case	count	share
$\sigma(n) = 2n$ (perfect)	4	0.000004000
$\sigma(n) > 2n$ (abundant)	247545	0.247545
$\sigma(n) < 2n$ (deficient)	752450	0.752451

Observation 2.2

this is the core. The equality holds for 4 numbers, the inequality just above for a quarter of all.The perfect numbers are 6, 28, 496 and 8128; the abundant ones number 247545, a ratio of 1 to 61886.2——on the very same σ, demanding equality drops the count by five orders of magnitude.

3. Around the Boundary, and the Odd Side

Computation 3.1: **the numbers with $\sigma(n)/2n$ in the band from 0.99 to 1.01 were counted.**

condition	count	ratio to the equality
$\sigma(n)/2n = 1$	4	1
$0.99 \le \sigma(n)/2n \le 1.01$	34212 (0.034212)	8553.0

Observation 3.2

the neighbourhood of the boundary is crowded.The one-percent band holds 34212 numbers, 8553.0 times those exactly on the boundary——the rarity of perfect numbers is not that σ avoids the neighbourhood of $2n$, but that "exactly" is demanded.

Computation 3.3: **the odd numbers were counted separately.**

odd case	count	smallest
abundant	1996	945
perfect	0	—

Observation 3.4

among odd numbers only the inequality appeared.There are 1996 odd abundant numbers starting at 945, yet not one odd perfect number below 10^{6}——it is not that odd numbers rarely exceed; they exceed, but they do not land exactly.

4. Building Numbers on the Equality

Computation 4.1: for each p with $2^p - 1$ prime, it was checked whether $N = 2^{p-1}(2^p - 1)$ satisfies $\sigma(N) = 2N$.

p	N	$\sigma(N) = 2N$
2	6	holds
7	8128	holds
13	33550336	holds
19	137438691328	holds
31	2305843008139952128	holds

> **Observation 4.2**
>
> numbers on the equality can be built without searching.All eight values $p = 2, 3, 5, 7, 13, 17, 19, 31$ gave $\sigma(N) = 2N$——being rare and having no recipe are different things.

5. The Separator

> **Observation 5.1**
>
> the separator is whether one asks for exact equality or for exceedance.On one and the same σ, the equality side holds 4 numbers (0.000004000) below 10^6 and the inequality side 247545 (0.247545)——an equality is a boundary, and a boundary itself carries no density.

> **Observation 5.2**
>
> placed beside Papers 247 and 300.Paper 247 showed that splitting by parity is not one phenomenon and cited the classification of even perfect numbers——here the split is not by parity but by equality against inequality.Paper 300 showed that sharing a root is decidable——the rarity and the density are two sides of one quantity.

Observation 5.3

what is not said.That odd perfect numbers do not exist, that Mersenne primes are infinite in number, and the exact value of the natural density are not treated——what is shown is that below 10^6 the two densities differ by five orders of magnitude.

6. Census and Closing

1. **4 perfect numbers (0.000004000)** (Computation 2.1): **6, 28, 496, 8128.**
2. **247545 abundant (0.247545)** (Computation 2.1): **the core.**
3. **752450 deficient (0.752451)** (Computation 2.1): **all the rest.**
4. **ratio 1 to 61886.2** (Observation 2.2): **five orders of magnitude.**
5. **34212 in the band (0.034212)** (Computation 3.1): **the boundary is crowded.**
6. **8553.0 times the equality** (Observation 3.2): **“exactly” is what makes it rare.**
7. **1996 odd abundant** (Computation 3.3): **smallest 945.**
8. **0 odd perfect** (Computation 3.3): **a fact below 10^6.**
9. **$N = 2305843008139952128$ at $p = 31$** (Computation 4.1): **they can be built.**
10. **equality or exceedance** (Observation 5.1): **the separator; Papers 247, 300.**

Closing: **Only four numbers sit exactly on the equality below a million, while the inequality just above it takes a quarter.$\sigma(n) = 2n$ holds for 6, 28, 496 and 8128 (0.000004000), while $\sigma(n) > 2n$ holds for 247545 (0.247545), a ratio of 1 to 61886.2——demanding equality drops the count by five orders of magnitude.The one-percent band around the boundary holds 34212 numbers (0.034212), 8553.0 times those on it, so the rarity is not that σ avoids the neighbourhood——it is that “exactly” is demanded.Among odd numbers there are 1996 abundant ones (smallest 945) but no perfect one below 10^6, and the form $2^{p-1}(2^p - 1)$ met the equality in all eight cases.The separator is whether one asks for exact equality or for exceedance.Placed among the earlier papers——standing beside Paper 247’s seat for even perfect numbers, and keeping Paper 300’s discipline in treating the two as sides of one quantity.To be honest——the count reaches only 10^6, and nothing is claimed about whether odd perfect numbers exist.**

References

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