# “One Tap Tells You Everything” Rests on Two Assumptions: Linearity and Time Invariance

— For a system with coefficients $[0.2, 0.3, 0.25, 0.15, 0.1]$, the impulse response alone predicts the response to a new input with error $0$ — the separator is whether the error moves when the amplitude changes or when the input is shifted — [Paper 813]

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September 2026

*— From thinking to syncing. —*

**Abstract**

**A linear time-invariant system is fixed completely by a single sequence, its impulse response.This paper breaks that assumption in two ways and measures how the prediction fails.No new theorem or law is claimed. First, with the assumptions in place, one tap does tell everything——for coefficients $[0.2, 0.3, 0.25, 0.15, 0.1]$ the prediction built from the impulse response differs from the actual response by $0$, and the DC gain equals the coefficient sum $1.000000$** (Section 2). **Second, breaking linearity makes it fail——clipping the amplitude at $0.60$ gives a relative error of $0.208758$** (Section 3). **Third, breaking time invariance makes it fail about as much——with coefficients drifting in time the relative error is $0.293023$, so the size of the error alone does not tell the two apart** (Section 3). **Fourth, and this is the core. They fail in different shapes——the clipping error moves with amplitude ($0.000000$ at $0.2$ times, $0.172705$ at $1$, $0.724235$ at $3$) while the drifting one stays at $0.379140$ at every amplitude** (Section 4). **Fifth, shifting reverses the roles——shifting the impulse by $5$ steps changes the response by $0$ under clipping and by $0.100000$ under drift ($0.146946$ at $10$ steps)** (Section 4). **Sixth, the separator is whether the error moves with amplitude or with a shift——“measure the impulse response and you know the system” can be said only after both operations have checked the assumptions** (Section 5). **Keywords**: impulse response, linearity, time invariance, convolution, superposition, DC gain.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——that a linear time-invariant system is fixed by its impulse response, convolution, superposition and time invariance are all standard.No real device is discussed——the clipping level $0.60$ and the coefficient swing of $0.5$ are values chosen to expose the failure.No general theory of nonlinear systems——expansions such as Volterra series, and system identification procedures, are not treated.The frequency side is not treated——transfer functions, phase and group delay are not entered; only the time side is examined.No noise——measurement error is not included; the only source of error is the broken assumption.Relation to earlier papers**: **Paper 520 showed that how many frequencies can be nulled is fixed by the number of coefficients, and its scope note stated "no filter is designed—requirements such as passband flatness, transition width and group delay are not entered"——this paper stands beside that seat: there how many coefficients there are, here the assumptions under which the coefficients fix everything.Paper 300 showed that whether two things share a root is decidable——a broken linearity and a broken time invariance are distinct roots even when the errors are of the same size, and changing the operation separates them.**What is added is **breaking the two assumptions separately and measuring the failure**, **counting how the failures swap between the amplitude operation and the shift operation**, **confirming that the size of the error alone cannot tell them apart**, and **placing the separator on amplitude versus shift**.

## 1. Notation and the Question

**Setting**: **when the output $y$ for an input $x$ can be written $y[n] = \sum_k h[k]x[n-k]$, the sequence $h$ is the impulse response.That form is available only when superposition (linearity) and invariance under a shift in time both hold.**

**Here $h = [0.2, 0.3, 0.25, 0.15, 0.1]$ (sum $1.000000$) is fixed and three systems compared: the system itself, the same system with its output clipped at $\pm 0.60$ (breaking only linearity), and the same system with coefficients drifting in time within $1 \pm 0.5$ (breaking only time invariance).**

**Question**: **when an assumption breaks, how does the prediction from the impulse response fail, and does the failure say which one broke?**

## 2. With the Assumptions in Place

**Computation 2.1**: **a single impulse was applied, its response recorded, and that alone used to predict the response to a random input.**

| measured | value |
|---|---|
| sum of coefficients | 1.000000 |
| steady output for a constant input of 1 | 1.000000 |
| largest difference between prediction and response | 0 |

> **Observation 2.2**
>
> **with the assumptions in place, one tap tells everything.The impulse response is only 5 numbers, yet it predicts the response to a 200-point input with error 0——that is the content of "one tap tells you everything".**

## 3. Breaking the Assumptions

**Computation 3.1**: **the same input was put through the clipped system and the drifting system and compared with the same prediction.**

| system | largest difference | relative to the signal peak |
|---|---|---|
| linear, time-invariant | 0 | 0 |
| clipped (linearity broken) | $1.252549 \times 10^{-1}$ | 0.208758 |
| drifting (time invariance broken) | $2.749734 \times 10^{-1}$ | 0.293023 |

> **Observation 3.2**
>
> **either break makes it fail, and the size alone does not tell them apart.The relative errors are 0.208758 and 0.293023, the same order——"the prediction failed" does not decide whether linearity or time invariance broke.**

## 4. The Shape of the Failure

**Computation 4.1**: **the input amplitude was set to 0.2, 1 and 3 times and the relative error measured again.**

| system | amplitude 0.2 | amplitude 1 | amplitude 3 |
|---|---|---|---|
| linear, time-invariant | 0.000000 | 0.000000 | 0.000000 |
| clipped | 0.000000 | 0.172705 | 0.724235 |
| drifting | 0.379140 | 0.379140 | 0.379140 |

**Observation 4.2**

**this is the core. The clipping error moves with amplitude; the drifting one does not.While the amplitude is small the clipped system errs by 0 (it never reaches the limit), and at 3 times it errs by 0.724235; the drifting system stays at 0.379140 throughout——a broken linearity is tied to "how large", a broken time invariance to "when it was applied".**

**Computation 4.3**: **the impulse was applied at later times and the shape of the response compared.**

| system | shifted by 5 | shifted by 10 |
|---|---|---|
| linear, time-invariant | 0 | 0 |
| clipped | 0 | 0 |
| drifting | 0.100000 | 0.146946 |

**Observation 4.4**

**under a shift the roles swap.The clipped system, which the amplitude test caught, passes the shift test with difference 0, while the drifting system fails it with 0.100000 and 0.146946——doing both operations decides which assumption broke.**

## 5. The Separator

**Observation 5.1**

**the separator is whether the error moves with amplitude or with a shift.The same “prediction fails” responds to the amplitude operation when linearity is broken and to the shift operation when time invariance is broken——“measure the impulse response and you know the system” comes after both checks.**

**Observation 5.2**

**placed beside Papers 520 and 300.Paper 520 showed that the number of nullable frequencies is fixed by the number of coefficients and stated that no filter is designed——here the assumptions under which those coefficients fix everything are measured instead.Paper 300 showed that sharing a root is decidable——two failures of similar size are distinct roots, separated by changing the operation.**

**Observation 5.3**

**what is not said.Real devices, a general theory of nonlinear systems, the frequency side and identification under noise are not treated——what is shown is that the two broken assumptions are separated by two operations.**

## 6. Census and Closing

1. **coefficient sum** 1.000000 (Computation 2.1): **equals the DC gain.**
2. **prediction error** 0 (Observation 2.2): **one tap suffices.**
3. **clipping gives** **0.208758** (Computation 3.1): **linearity broken.**
4. **drift gives** **0.293023** (Computation 3.1): **time invariance broken.**
5. **size does not tell them apart** (Observation 3.2): **same order.**
6. **0.724235 at amplitude** 3 (Computation 4.1): **the core.**
7. **drift stays at** **0.379140** (Computation 4.1): **amplitude does not matter.**
8. **shift by** 5 **gives** **0.100000** (Computation 4.3): **tied to time.**
9. **shift by** 10 **gives** **0.146946** (Computation 4.3): **clipping stays at** 0**.**

10. **amplitude or shift** (Observation 5.1): **the separator; Papers 520, 300.**

**Closing**: **“One tap tells you everything” rests on two assumptions: linearity and time invariance.For coefficients $[0.2, 0.3, 0.25, 0.15, 0.1]$ the prediction from the impulse response is exact and the DC gain equals the coefficient sum $1.000000$——five numbers fix a $200$-point response.Clipping the amplitude gives a relative error of $0.208758$ and drifting coefficients give $0.293023$, so the size alone cannot tell them apart——one has to change the operation.The clipping error moves with amplitude ($0.000000$ at $0.2$ times, $0.724235$ at $3$) while the drifting one is $0.379140$ at every amplitude, and shifting the impulse by $5$ steps gives $0$ for clipping and $0.100000$ for drift.The separator is whether the error moves with amplitude or with a shift.Placed among the earlier papers——standing beside Paper 520’s seat for not designing filters, and keeping Paper 300’s discipline in treating the two failures as distinct roots.To be honest——only two ways of breaking were tried, and how much either matters in a real device was not measured.**

## References


1. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
2. Yamagishi, Y. (2026). *How Many Frequencies Can Be Nulled Is Fixed by the Number of Coefficients* [Paper 520]. Zenodo. 10.5281/zenodo.22862501


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