The Distance to the Moon Comes Out of a Shadow and a Clock, but Ignoring the Shadow's Taper Puts It Forty Percent Too Far

— using only that the umbra looks **2.60** times the Moon's diameter and that the Moon subtends **0.518** degrees, a cylindrical shadow gives **542072.9** km, **1.410179** times the true value — the separator is whether the shadow is taken as the size of the Earth or as a narrowing cone — [Paper 814]

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— *From thinking to syncing.* —

Abstract

The distance to the Moon can be had without measuring any parallax, from the size of the shadow during a lunar eclipse and a clock.This paper carries out that calculation two ways—taking the shadow as a cylinder and as a narrowing cone—and counts how far each lands from the true value.No new theorem or law is claimed. First, taking the shadow as the size of the Earth puts the Moon forty percent too far——542072.9 km, which is 1.410179 times the true 384400 km (Section 2). **Second, and this is the core. Allowing for the taper set by the Sun's angular diameter lands within one percent——388373.1 km (1.010336 times), a factor of 1.395753 from the other answer** (Section 3). **Third, a clock alone gives the same answer——an umbral passage of 206 minutes at 1.022 km/s gives a lunar diameter of 3508.9 km and a distance of 388114.4 km (1.009663 times)** (Section 4). **Fourth, the same geometry also gives the Moon's size——a radius of 1755.6 km, which is 1.010477 times the true 1737.4 km** (Section 3). **Fifth, no parallax is needed——all that is used is the Earth's diameter, the Moon's angular diameter, the ratio of shadow to Moon, and a clock** (Section 5). **Sixth, the separator is whether the shadow is taken as the size of the Earth or as a narrowing cone——one and the same observation yields answers forty percent apart** (Section 5). **Keywords**: lunar eclipse, umbra, angular diameter, distance to the Moon, geometry, representative values.

Scope of this paper (scope note): **No new theorem or law is claimed——the geometry of a lunar eclipse, the narrowing of the umbra by the Sun's angular diameter, and the Moon's angular diameter and orbital speed are all known.The values are representative——the shadow-to-Moon ratio 2.60, the angular diameters 0.518 and 0.533 degrees, the umbral passage of 206 minutes and the speed 1.022 km/s all vary from eclipse to eclipse.The atmosphere is left out ——the few-percent enlargement of the umbra by the Earth's atmosphere is not included.The orbit is taken as circular——changes of distance and angular size with eccentricity are not included; only mean values are used.History is not adjudicated——who measured what, and which ancient value was right, are not treated.Parallax is not used ——measuring from two places at once is not entered.Relation to earlier papers**: **Paper 806 showed that a transit converts the Sun's parallax, too small to measure as an angle, into a difference of times through the parallax of Venus ——this paper stands beside that seat: there the road that uses parallax, here the road that uses none.Paper 805 separated reading longitude by a clock from reading it by the Moon, and its scope note stated that corrections for parallax and refraction are not entered——here too no corrections are entered; only the geometry of the shadow.Paper 297 showed that cosmic distances form a seven-rung ladder of which only the first is measured directly ——the distance to the Moon sits at the foot of that ladder.**What is added is **counting that the same observation gives answers forty percent apart depending on the taper**, **confirming that the clock-only road lands at 1.009663 times**, **showing that the same geometry gives the Moon's radius**, and **placing the separator on cylinder versus cone**.

1. Notation and the Question

Setting: **during a lunar eclipse the Moon passes through the umbra cast by the Earth.The shadow looks about 2.60 times the Moon's diameter, and the Moon subtends 0.518 degrees.**

Writing D_{shadow} for the shadow's diameter, D_{Moon} for the Moon's and d for the distance, the angular sizes give

$$D_{\text{shadow}} = d\,\theta_{\text{shadow}}, \qquad \theta_{\text{shadow}} = 2.60 \times \theta_{\text{Moon}}, \qquad \theta_{\text{Moon}} = 0.518^\circ$$

Question: **how much does the distance change with what the shadow's diameter is taken to be?**

2. Taking the Shadow as a Cylinder

Computation 2.1: **if the shadow never narrows, its diameter equals the Earth's, 12742 km.**

$$d = \frac{D_{\text{Earth}}}{\theta_{\text{shadow}}} = \frac{12742}{2.60 \times 0.518^\circ} = 542072.9\ \text{km}$$

Observation 2.2

this puts the Moon forty percent too far.It is 1.410179 times the true 384400 km——taking the shadow as a cylinder makes it wider than it is, and the distance stretches to cover that width.

3. Taking the Shadow as a Narrowing Cone

Computation 3.1: **the Sun is not a point but subtends 0.533 degrees, so the umbra narrows with distance.At distance d its diameter is the Earth's diameter less $d\theta_{\text{Sun}}$.**

$$d\,\theta_{\text{shadow}} = D_{\text{Earth}} - d\,\theta_{\text{Sun}} \quad\Longrightarrow\quad d = \frac{D_{\text{Earth}}}{\theta_{\text{shadow}} + \theta_{\text{Sun}}} = 388373.1\ \text{km}$$

how the shadow is taken	distance (km)	ratio to the true value
cylinder (no taper)	542072.9	1.410179
cone (tapering)	388373.1	1.010336

Observation 3.2

this is the core. With the taper the answer lands within one percent.The two answers differ by a factor of 1.395753, the untapered one being the farther——the Sun's angular diameter is about the same as the Moon's, so its effect is not a small correction but moves the answer by forty percent.

Computation 3.3: **the same distance and angular diameter give the Moon's radius.**

$$r_{\text{Moon}} = \frac{d\,\theta_{\text{Moon}}}{2} = 1755.6\ \text{km}$$

That is 1.010477 times the true 1737.4 km.

4. Measuring With a Clock Alone

Computation 4.1: **from first to last umbral contact the Moon travels the shadow's diameter plus its own, that is 3.60 lunar diameters.A passage of 206 minutes at 1.022 km/s gives 12631.9 km.**

measured	value
distance travelled	**12631.9** km
lunar diameter (divide by **3.60**)	**3508.9** km
distance (divide by the angular diameter)	**388114.4** km
ratio to the true value	**1.009663**

> **Observation 4.2**
>
> **a clock alone gives the same answer.The road that measures the shadow as an angle and the road that measures the eclipse as a duration agree at 1.010336 and 1.009663——both read the same geometry through a different quantity.**

5. The Separator

> **Observation 5.1**
>
> **the separator is whether the shadow is taken as the size of the Earth or as a narrowing cone.One and the same observation—shadow 2.60 times the Moon, Moon 0.518 degrees—gives 542072.9 km one way and 388373.1 km the other——what makes the difference is not the observation but that single choice.**

> **Observation 5.2**
>
> **no parallax is needed.All that is used is the Earth's diameter, the Moon's angular diameter, the ratio of shadow to Moon, and a clock; no simultaneous view from two places is required——one observer with one night suffices.**

Observation 5.3

placed beside Papers 806, 805 and 297.Paper 806 turned a parallax into a difference of times, Paper 805 read longitude by clock and by Moon, and Paper 297 counted the rungs of the distance ladder——here the foot of that ladder is reached without parallax.

Observation 5.4

what is not said.The enlargement of the umbra by the atmosphere, the eccentricity of the orbit, and the merits of historical measurements are not treated——what is shown is that how the shadow is taken moves the answer by forty percent.

6. Census and Closing

1. **cylinder gives 542072.9 km** (Computation 2.1): **1.410179 times.**
2. **cone gives 388373.1 km** (Computation 3.1): **the core; 1.010336 times.**
3. **ratio 1.395753** (Observation 3.2): **forty percent apart.**
4. **lunar radius 1755.6 km** (Computation 3.3): **1.010477 times.**
5. **travel 12631.9 km** (Computation 4.1): **the clock road.**
6. **lunar diameter 3508.9 km** (Computation 4.1): **divide by 3.60.**
7. **clock gives 388114.4 km** (Computation 4.1): **1.009663 times.**
8. **the two roads agree** (Observation 4.2): **one geometry, two quantities.**
9. **no parallax** (Observation 5.2): **one observer suffices.**
10. **cylinder or cone** (Observation 5.1): **the separator; Papers 806, 805, 297.**

Closing: **The distance to the Moon comes out of a shadow and a clock, but ignoring the shadow's taper puts it forty percent too far.Using only that the umbra looks 2.60 times the Moon's diameter and that the Moon subtends 0.518 degrees, taking the shadow as the Earth's diameter gives 542072.9 km, 1.410179 times the true value——a shadow taken wider than it is stretches the distance.Allowing for the narrowing set by the Sun's 0.533 degrees gives 388373.1 km (1.010336 times), the two differing by 1.395753——the Sun's angular diameter is about the Moon's own, so this is no small correction.The clock road gives a lunar diameter of 3508.9 km and a distance of 388114.4 km (1.009663 times) from a 206-minute umbral passage, and the same geometry gives a lunar radius of 1755.6 km.The separator is whether the shadow is taken as the size of the Earth or as a narrowing cone.Placed among the**

earlier papers——standing beside Paper 806's parallax seat, entering no corrections as in Paper 805, and reaching the foot of Paper 297's ladder without parallax.To be honest——every value used is representative, and neither the atmospheric enlargement of the umbra nor the eccentricity is included.

References

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