

The Size of the Core Is Fixed by the P-Wave Shadow, and That It Is Liquid by the Absence of S Waves

— a core radius from the P-wave shadow with straight rays comes out **1.139667** times the true one, while the S-wave speed $\sqrt{\mu/\rho}$ is exactly **0.000000** at zero shear modulus — the separator is whether one asks the size or the state — [Paper 824]

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September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/824>

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Abstract

Two things are known about the Earth's core: how big it is and that it is liquid. This paper counts that each comes from a different observation and that neither yields the other. No new theorem or law is claimed. First, the size is fixed by the shadow: taking the shadow to begin at 103 degrees with straight rays gives a radius of 3966.0 km, **1.139667** times the true value (Section 2). Second, straight rays make the core too large: the true core would put the shadow at **113.783157** degrees, **10.783157** beyond the observed 103 (Section 2). Third, and this is the core. The state is fixed by a different wave: the S-wave speed is $\sqrt{\mu/\rho}$, exactly **0.000000** when the shear modulus is 0 (Section 3). Fourth, P waves still travel in the same place: at $\mu = 0$ they run at **8.062258** km/s (Section 3). Fifth, a soft solid cannot account for the absence: $\mu = 10^9$ Pa would carry S waves at **0.316228** km/s and 10^{10} Pa at **1.000000** km/s (Section 3). Sixth, the separator is whether one asks the size or the state: the angle of the shadow speaks only of size, and the absence of shear waves only of state (Section 4). **Keywords:** seismic waves, shadow zone, outer core, shear modulus, shear wave, representative values.

Scope of this paper (scope note): No new theorem or law is claimed: the P-wave shadow zone, the failure of S waves to cross the outer core, and the elastic wave speeds are all known. The treatment is coarse: rays are taken as straight for the size estimate; no travel-time curve or velocity structure is built. The values are representative: a shadow from 103 to 143 degrees, an outer-core density of 10000 kg/m³ and a bulk modulus of 6.5×10^{11} Pa are representative. The inner core is not entered: that it is solid, and the waves that cross it, are not treated. Observation is not discussed: networks, hypocentre determination and phase identification are not the subject. Composition is not discussed: whether the core is iron is not treated. Relation to earlier papers: Paper 814 showed that the distance to the Moon comes out of a shadow and a clock, but that ignoring the taper puts it forty percent too far. This paper stands beside it: there a shadow taken as a cylinder, here rays taken as straight, and in both the assumption moves the answer. Paper 300 showed that whether two things share a root is decidable: the observation fixing the size and the one fixing the state are distinct roots, and neither yields the other. What is added is deriving the core radius from the shadow and measuring the ratio 1.139667, measuring the 10.783157-degree discrepancy, listing the S-wave speed against the shear modulus, and placing the separator on size versus state.

1. Notation and the Question

Setting: seismic waves come as P waves, compressing along the direction of travel, and S waves, shearing across it. Their speeds are

$$v_P = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}, \quad v_S = \sqrt{\frac{\mu}{\rho}}$$

with K the bulk modulus, μ the shear modulus and ρ the density. The Earth's radius is taken as 6371 km and the outer core's as 3480 km.

Question: what fixes the size of the core, and what fixes that it is liquid?

2. Size From the Shadow

Computation 2.1: with straight rays the grazing ray reaches an epicentral distance of $2 \arccos(r/R)$. Putting the observed shadow onset of 103 degrees into this gives the core radius.

measured	value
shadow onset (representative)	103 degrees
core radius implied	3966.0 km
ratio to the true radius	1.139667
onset implied by the true core	113.783157 degrees
discrepancy	10.783157 degrees

Observation 2.2

straight rays make the core too large.It comes out 1.139667 times the true 3480 km: speeds rise with depth, so rays curve upward and the shadow begins nearer than straight rays predict; assuming straight rays pays for that difference with a larger core.

Observation 2.3

the order of magnitude still comes out.The shadow zone runs from 103 to 143 degrees, a span of 40, and one angle gives the size to about ten percent: the angle speaks not of “something is in there” but of “this big”.

3. State From the Shear Wave

Computation 3.1: with an outer-core density of 10000 kg/m³ and a bulk modulus of 6.5×10^{11} Pa, the shear modulus was varied.

μ (Pa)	P wave (km/s)	S wave (km/s)
0	8.062258	0.000000
10^9	8.070522	0.316228
10^{10}	8.144528	1.000000
10^{11}	8.850612	3.162278

Observation 3.2

this is the core. Only at $\mu = 0$ is the S-wave speed exactly zero. In the same place P waves still run at 8.062258 km/s: that shear waves do not pass means there is nothing there to resist shear, and that is the content of “it is liquid”.

Observation 3.3

a soft solid cannot account for it. At $\mu = 10^9$ Pa S waves would run at 0.316228 km/s and at 10^{10} Pa at 1.000000 km/s: “soft, hence slow” would show a weak arrival, not a shadow; the observation states an absence.

4. The Separator

Observation 4.1

the separator is whether one asks the size or the state. The angle of the shadow speaks only of size and the absence of shear waves only of state: neither observation yields the other.

Observation 4.2

placed beside Papers 814 and 300. Paper 814 showed that taking an eclipse shadow as a cylinder puts the Moon forty percent too far. Here taking rays as straight makes the core 1.139667 times too large: the same shape, where an assumption moves the answer. Paper 300 showed that sharing a root is decidable: size and state are distinct roots.

Observation 4.3

what is not said. Travel-time curves, velocity structure, the inner core, observation networks and composition are not treated. What is shown is that two questions need two waves.

5. Census and Closing

1. shadow onset **103 degrees** (Computation 2.1): **representative observation**.
2. core radius **3966.0 km** (Computation 2.1): **with straight rays**.
3. ratio **1.139667** (Observation 2.2): **about ten percent large**.
4. true core implies **113.783157 degrees** (Computation 2.1): **beyond the observation**.
5. discrepancy **10.783157 degrees** (Computation 2.1): **the curvature of rays**.
6. shadow spans **40 degrees** (Observation 2.3): **103 to 143**.
7. S wave **0.000000** at $\mu = 0$ (Computation 3.1): **the core**.
8. P wave **8.062258 km/s** there (Computation 3.1): **compression still passes**.
9. **1.000000 km/s** at $\mu = 10^{10}$ Pa (Observation 3.3): **a soft solid will not do**.
10. size or state (Observation 4.1): **the separator; Papers 814, 300**.

Closing: The size of the core is fixed by the P-wave shadow, and that it is liquid by the absence of S waves. Taking the shadow onset at 103 degrees with straight rays gives a radius of 3966.0 km, 1.139667 times the true value, while the true core would put the onset at 113.783157 degrees, 10.783157 beyond the observation: rays curve because speed rises with depth, and the straight assumption pays for it with a larger core. The S-wave speed is $\sqrt{\mu/\rho}$ and is exactly 0.000000 only when the shear modulus vanishes, while P waves still run at 8.062258 km/s in the same place; a shear modulus of 10^{10} Pa would carry S waves at 1.000000 km/s, so the observation states an absence, not a slowness. The separator is whether one asks the size or the state. Placed among the earlier papers: standing beside Paper 814's "the assumption moves the answer", and keeping Paper 300's discipline between size and state. To be honest. This is a coarse calculation with straight rays, treating neither travel-time curves nor the inner core.

References

1. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. YOI: <https://www.yamagishiyuuki.com/yoi/300>; DOI: 10.5281/zenodo.22171914
2. Yamagishi, Y. (2026). *The Distance to the Moon Comes Out of a Shadow and a Clock, but Ignoring the Shadow's Taper Puts It Forty Percent Too Far* [Paper 814]. YOI: <https://www.yamagishiyuuki.com/yoi/814>; DOI: 10.5281/zenodo.23015478

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