A Satellite Tells You Not the Planet's Mass but the Sum of the Planet's and the Satellite's

— A semi-major axis of 3.844×10^8 m and a period of 27.321661 days give $M + m$, not the mass of the Earth

— the separator is whether one measures a sum or a ratio — [Paper 833]

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— *From thinking to syncing.* —

Abstract

It is said that a satellite's period gives the planet's mass.This paper counts that what it gives is a sum, not the planet's mass.No new theorem or law is claimed. First, Kepler's third law returns a sum——$4\pi^2 a^3/(GT^2)$ is $M + m$, not M (Section 2). **Second, and this is the core. The difference is large enough to matter——for the Earth and the Moon the sum exceeds the Earth by 0.012151, which is 4.47 times the 0.002716 caused by rounding the axis** (Section 2). **Third, the difference is set by the companion's weight——1.661031×10^{-8} for Mars and Phobos, 9.534422×10^{-4} for the Sun and Jupiter, 1.215087×10^{-2} for the Earth and the Moon, 1.085112×10^{-1} for Pluto and Charon** (Section 2). **Fourth, splitting the sum needs another measurement——the barycentre lies 4670.8 km from the Earth's centre, 0.733133 of the Earth's radius, inside the Earth** (Section 3). **Fifth, for a planet without a satellite that other measurement is missing——Venus's share of the sum is 2.447302×10^{-6}, and the period shifts by only 23.7535 seconds in 224 days** (Section 3). **Sixth, the separator is whether one measures a sum or a ratio——the period returns the sum, the barycentre returns the ratio; only together do they fix a mass** (Section 4). **Keywords**: Kepler's third law, two-body problem, sum of masses, barycentre, planetary mass, model.

Scope of this paper (scope note): **No new theorem or law is claimed——that Kepler's third law carries the sum of the two masses, that the barycentre divides the separation in the ratio of the masses, and that the masses of satellite-less planets were obtained from spacecraft flybys and from perturbations of other planets, are all known.No observational accuracy is claimed——the inputs are representative values for reading orders of magnitude, and the value of the sum itself moves by 0.2716 per cent with the rounding of the axis.The history of measurement is not discussed——who measured what, and when, is not treated.More than two bodies are not treated——obtaining masses from perturbations is only named.No relativistic correction is included——perihelion precession and time dilation are not the subject.Relation to earlier papers**: **Paper 685 showed that the name "asteroid belt" overstates what is there by two orders of magnitude, and its scope note placed "the masses of asteroids and estimates of the belt's total mass" among the known——this paper does not sit in that seat but stands beside it: there the total of a belt, here how one planet's mass is measured.Paper 28 showed that the exponent $3/2$ is not a dimension——there the identity of the exponent in the same law, here the identity of the quantity the law returns.**What is added is **measuring the gap 0.012151 against the rounding discrepancy 0.002716 and finding it 4.47 times as large**, **laying the gap out over four pairs**, **measuring the barycentre at 0.733133 of the Earth's radius**, and **placing the separator on sum versus ratio**.

1. Notation and the Question

Setting: **let the planet's mass be M, the satellite's m, the semi-major axis a and the period T.**

$$\frac{a^3}{T^2} = \frac{G(M+m)}{4\pi^2}, \qquad M + m = \frac{4\pi^2 a^3}{G T^2}$$

On the right, M and m enter only as a sum.Which of them is how much does not come out of this equation.

Question: **what, exactly, has been measured?**

2. The Sum and the Planet's Mass

Computation 2.1: $a = 3.844 \times 10^{8}$ **m and** $T = 27.321661$ **days were put in.**

quantity	value
what the formula returns, $M + m$	6.029239×10^{24} kg
the known $M + m$	6.045660×10^{24} kg
discrepancy (relative)	0.002716
the Earth's mass	5.972200×10^{24} kg
sum minus Earth (relative)	0.012151

> **Observation 2.2**
>
> **this is the core. The gap is not buried in the rounding.The gap 0.012151 is 4.47 times the discrepancy 0.002716 caused by rounding the axis——however carefully the inputs are prepared, calling $M + m$ by the name M leaves an error behind.**

Computation 2.3: $m/(M + m)$ **was computed for four pairs.**

pair	m/M	$m/(M + m)$
Mars and Phobos	1.661031×10^{-8}	1.661031×10^{-8}
Sun and Jupiter	9.543521×10^{-4}	9.534422×10^{-4}
Earth and Moon	1.230032×10^{-2}	1.215087×10^{-2}
Pluto and Charon	1.217191×10^{-1}	1.085112×10^{-1}

> **Observation 2.4**
>
> **the gap is set by the companion's weight.With Phobos it vanishes eight orders down; with Charon it exceeds a tenth——"if the satellite is light the sum is close to the primary" is true, but how close differs from satellite to satellite.**

3. The Measurement That Splits the Sum

Computation 3.1: **the barycentre divides the separation in the ratio of the masses.**

$$r = a\,\frac{m}{M+m}$$

quantity	value
Earth--Moon barycentre	**4670.8** km from the Earth's centre
the Earth's radius	**6371.0** km
barycentre / radius	**0.733133**

> **Observation 3.2**
>
> **the barycentre returns a ratio.The Earth--Moon barycentre sits at 0.733133 of the Earth's radius, that is, inside the Earth——the period returns the sum and the barycentre the ratio, and only together do they fix M and m separately.**

Computation 3.3: **a planet without a satellite leaves only its motion about the Sun. Venus was tried.**

quantity	value
Venus's share of the sum	2.447302×10^{-6}
period	**224.6752** days
period with Venus's mass ignored	**224.6755** days
shift	**23.7535** seconds

> **Observation 3.4**
>
> **when the share is small, it cannot be extracted from the sum.Discarding Venus's mass entirely moves a 224-day period by 23.7535 seconds——which is why the masses of satellite-less planets were obtained from flybys and from perturbations of other planets.**

Computation 3.5: **conversely, mistaking the sum for the Earth’s mass alone moves the Moon’s period.**

quantity	value
correct period	**27.284530** days
period with the Moon’s mass ignored	**27.451821** days
shift	**4.0150** hours

> **Observation 3.6**
>
> **the same mistake costs 23.7535 seconds at Venus and 4.0150 hours at the Moon.The shares differ by more than three orders——whether something “may be ignored” is decided not by the formula but by the companion’s weight.**

4. The Separator

> **Observation 4.1**
>
> **the separator is whether one measures a sum or a ratio.The period and the axis return only a sum; the barycentre returns only a ratio——neither alone fixes a mass.**

> **Observation 4.2**
>
> **placed beside Paper 685.685 showed that the name “asteroid belt” overstates what is there by two orders of magnitude——there a name inflates a quantity, here the quantity a formula returns is itself slightly larger. Both restate what was measured.Paper 28 showed that the exponent $3/2$ is not a dimension——the identity of the exponent, and the identity of the returned quantity, in the same law.**

> **Observation 4.3**
>
> **what is not said.Observational accuracy, the history of measurement, perturbations among three or more bodies and relativistic corrections are not treated——what is shown is that the quantity returned by $4\pi^2 a^3/(GT^2)$ is named $M+m$, not M.**

5. Census and Closing

1. **the formula returns $M+m$** (Section 1): **they enter only as a sum.**
2. **the sum is 6.029239×10^{24} kg** (Computation 2.1): **the Earth is 5.972200×10^{24} kg.**
3. **the gap is 0.012151** (Computation 2.1): **the core.**
4. **the rounding gives 0.002716** (Computation 2.1): **the gap is 4.47 times that.**
5. **Charon gives 1.085112×10^{-1}** (Computation 2.3): **above a tenth.**
6. **the barycentre is 4670.8 km** (Computation 3.1): **the measurement that returns a ratio.**
7. **0.733133 of the radius** (Computation 3.1): **inside the Earth.**
8. **Venus's share is 2.447302×10^{-6}** (Computation 3.3): **23.7535 seconds.**
9. **the Moon gives 4.0150 hours** (Computation 3.5): **the same mistake, other orders.**
10. **sum or ratio** (Observation 4.1): **the separator; Papers 685, 28.**

Closing: **A satellite tells you not the planet's mass but the sum of the planet's and the satellite's.The axis and the period give $4\pi^2a^3/(GT^2)=M+m$, and for the Earth and the Moon the sum exceeds the Earth by 0.012151, which is 4.47 times the 0.002716 from rounding the axis——because M and m enter the formula only as a sum.Splitting the sum needs a second measurement, the barycentre, which lies 4670.8 km from the Earth's centre, 0.733133 of the Earth's radius, inside the Earth——because the barycentre returns a ratio.At satellite-less Venus, discarding the planet's mass entirely moves a 224-day period by only 23.7535 seconds, which is why another measurement is needed.Placed among the earlier papers——beside Paper 685's "a name inflates a quantity", and alongside Paper 28's identity of the exponent, counting the identity of the returned quantity.To be honest——the inputs used here are representative values for reading orders of magnitude, and the value of the sum itself moves by 0.2716 per cent with the rounding.**

References

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