Two Mirror Images Return the Same Numbers However Many Distances You Measure

— A tetrahedral centre carrying four different groups and its mirror image have all six interatomic distances equal to **1.632993161855**, with difference **0**, and all three inertia eigenvalues equal to **2.666666666667**, again with difference **0** — the separator is whether one measures distance or orientation — [Paper 836]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— From thinking to syncing. —

Abstract

A left hand and a right hand have the same shape and yet do not superimpose.This paper counts which quantity separates that “same” from that “different”.No new theorem or law is claimed. First, every distance is the same——all six interatomic distances are 1.632993161855, and the difference between the two mirror images is 0 (Section 2). **Second, quantities built from distances are the same too——all three inertia eigenvalues are 2.666666666667, difference 0** (Section 2). **Third, and this is the core. Only the signed volume reverses——+0.513200239280 against −0.513200239280, summing to 0** (Section 2). **Fourth, a partner that receives three points can tell them apart——the mirror image can be turned onto only two, leaving the third off by the edge length 1.632993** (Section 3). **Fifth, spreading the error does not remove it——over all rotations the root mean square never falls below 0.669123** (Section 3). **Sixth, the separator is whether one measures distance or orientation——mixed in equal parts the signed volumes cancel to 0, and the mixture no longer carries the orientation** (Section 4). **Keywords**: enantiomer, chirality, signed volume, three-point attachment, racemic mixture, model.

> **Scope of this paper (scope note): No new theorem or law is claimed——that enantiomers are non-superimposable, that in an achiral environment their physical properties agree, that telling them apart requires a partner that is itself not mirror-symmetric (a chiral reagent, a receptor, polarised light), and the phrase “three-point attachment”, are all known.No real molecule is claimed——the four points of the tetrahedron are a model; neither the bond lengths nor the angles are real values.Chemical reactions are not treated——asymmetric synthesis and optical resolution are not entered.Left-right asymmetry in the body is not treated——the position of the heart and situs inversus are not the subject.Origins are not discussed——why life chose one mirror image is not treated.No value of optical rotation is given——which substance turns light by how much is not stated.Relation to earlier papers: Paper 300 showed that whether two things share a root is decidable——this paper stands beside it: here too “same or different” can be answered only once one fixes which quantity to look at.Paper 194 showed that the four means are one family with a single equality fence——0.942809 appears in its table too, but there as a ratio of means, a different source from the root mean square here (they are not called a shared root).**What is added is **confirming that the six distances and the three inertia eigenvalues both differ by 0, measuring the signed volume at ± 0.513200239280 with sum 0, measuring that the mirror image fits only two points, leaving the third off by 1.632993, and that spreading leaves 0.669123**, and **placing the separator on distance versus orientation**.

1. Notation and the Question

Setting: four different groups A, B, C, D are placed along four directions from a centre.The directions are the vertices of a regular tetrahedron, and the mirror image is made by reversing the sign of one coordinate.

$$A = \tfrac{1}{\sqrt{3}}(1,1,1), \quad B = \tfrac{1}{\sqrt{3}}(1,-1,-1), \quad C = \tfrac{1}{\sqrt{3}}(-1,1,-1), \quad D = \tfrac{1}{\sqrt{3}}(-1,-1,1)$$

The signed volume is the determinant of A, B, C seen from D, divided by 6.A distance is the straight length between two atoms.

Question: by which quantity do two mirror images differ?

2. What Returns the Same Numbers

Computation 2.1: the six interatomic distances were laid out for the molecule and its mirror image.

pair	molecule	mirror image	difference
A-B	1.632993161855	1.632993161855	0
A-C	1.632993161855	1.632993161855	0
A-D	1.632993161855	1.632993161855	0
B-C	1.632993161855	1.632993161855	0
B-D	1.632993161855	1.632993161855	0
C-D	1.632993161855	1.632993161855	0

> **Observation 2.2**
>
> **distances do not separate them.All six are the same number to the last digit——because a mirror does not change lengths.**

Computation 2.3: **the inertia eigenvalues were computed.**

quantity	molecule	mirror image
first eigenvalue	2.666666666667	2.666666666667
second eigenvalue	2.666666666667	2.666666666667
third eigenvalue	2.666666666667	2.666666666667

> **Observation 2.4**
>
> **quantities built from distances do not separate them either.Inertia is built from squared distances, so it survives the mirror unchanged——the three being equal follows from the symmetry of the tetrahedron (in this model it looks like a sphere).**

Computation 2.5: **the signed volume was computed.**

quantity	value
molecule	$+0.513200239280$
mirror image	-0.513200239280
sum	0

> **Observation 2.6**
>
> **this is the core. This one quantity reverses.The magnitude is the same, only the sign flips, and applying the mirror twice restores it——because distances pass through a square and drop the sign, while a determinant does not.**

3. Receiving Three Points

Computation 3.1: **three sockets were built at the positions of A, B and C of the molecule, and the mirror image was turned onto them.**

measured	offset
A	0.000000000000
B	0.000000000000
C	1.632993161855

> **Observation 3.2**
>
> **two points always fit.The distances from the centre and the angle between the two points agree, so a rotation carrying two onto two must exist——and then the third lands a full edge away, on a neighbouring socket.**

Computation 3.3: **all rotations were tried, minimising the root mean square over the three points.**

what is fitted	root mean square
the molecule (unturned)	0.000000000000
mirror image, two points matched	0.942809
mirror image, best over all rotations	0.669123

> **Observation 3.4**
>
> **spreading is better, but never 0.Matching two gives 0.942809; spreading over three brings it to 0.669123 ——either way it stands away from 0, unlike the molecule's 0.000000000000.**

> **Observation 3.5**
>
> **a socket with only two points cannot tell them apart.Two points fit at 0, so to a partner that sees only two the mirror images look alike——the third point is what does the telling.**

4. The Separator

> **Observation 4.1**
>
> **the separator is whether one measures distance or orientation (the signed volume).However many distances are measured they return the same numbers, and only the signed volume reverses——and to read a signed volume, the reader must itself not be mirror-symmetric.**

> **Observation 4.2**
>
> **mixed in equal parts, the orientation becomes unreadable.The mean signed volume is 0——each component carries it, and the mixture does not.**

Observation 4.3

placed beside Paper 300.300 showed that whether two things share a root is decidable——here too "same or different" can be answered only once one fixes which quantity to look at: by distance the same, by orientation reversed.With Paper 194 this paper shares 0.942809 but not a root——there a ratio of four means, here a root mean square of three offsets.

Observation 4.4

what is not said.No real molecule, no reaction, no bodily asymmetry, no origin and no value of optical rotation is treated——what is shown is that a single determinant carries the whole distinction.

5. Census and Closing

1. **six distances at 1.632993161855** (Computation 2.1): **difference 0.**
2. **inertia eigenvalues 2.666666666667** (Computation 2.3): **all three alike.**
3. **signed volume ± 0.513200239280** (Computation 2.5): **the core.**
4. **sum 0** (Computation 2.5): **same magnitude.**
5. **two points fit at 0.000000000000** (Computation 3.1): **a rotation exists.**
6. **the third is off by 1.632993** (Computation 3.1): **a full edge.**
7. **matching two gives 0.942809** (Computation 3.3): **over three points.**
8. **the best is 0.669123** (Computation 3.3): **spreading never reaches 0.**
9. **mixed, the mean is 0** (Observation 4.2): **orientation unreadable.**
10. **distance, or orientation** (Observation 4.1): **the separator; Papers 300, 194.**

Closing: **Two mirror images return the same numbers however many distances you measure.All six interatomic distances are 1.632993161855 with difference 0, and all three inertia eigenvalues are 2.666666666667, again with difference 0——because distances pass through a square and drop the mirror's sign.The only thing that reverses is the signed volume, $+0.513200239280$ against -0.513200239280, summing to 0——because a determinant does not drop the sign.A partner receiving three points can tell them apart, for the mirror image fits only two, leaving the third off by the edge length 1.632993, and the best over all rotations still leaves a root mean square of 0.669123 .Placed among the earlier papers——beside Paper 300's rule that "same or different" waits on choosing a quantity,**

and calling the 0.942809 shared with Paper 194 a distinct root.To be honest——the four points of the tetrahedron are a model, and neither the bond lengths nor the angles are those of a real molecule.

References

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