Two-Point Discrimination Has Two Limits: Narrowing the Receptive Field Never Beats the Receptor Spacing

— With receptors packed densely the threshold is proportional to the receptive-field width, **2.286668** times σ (spread 2.008×10^{-4}), and however far the dip criterion is loosened it never falls below 2σ — the separator is whether one asks about blur or about spacing — [Paper 837]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— From thinking to syncing. —

Abstract

The fineness of touch is explained both by small receptive fields and by many receptors.This paper counts that these are two different limits, and that improving one alone stops.No new theorem or law is claimed. First, with dense receptors the threshold is set by the receptive-field width——it is 2.286668 times σ, and sweeping σ over a factor of 8 leaves a spread of only 2.008×10^{-4} (Section 2). **Second, that limit has a floor——loosening the dip criterion from 0.20 to 0.01 brings the threshold only from 2.659331 to 2.120562, never below 2σ** (Section 2). **Third, and this is the core. Narrowing the field leaves the spacing behind——at a spacing of 2.0, narrowing σ to a fortieth moves the threshold only from 9.313223 to 3.000513** (Section 3). **Fourth, the floor is written in the spacing——it is 1.500257 times the spacing** (Section 3). **Fifth, that floor moves with where the two points land——from 1.003722 to 2.000256 times the spacing, and the landing place cannot be chosen** (Section 3). **Sixth, the separator is whether one asks about blur or about spacing——one threshold holds two limits, and the larger decides** (Section 4). **Keywords**: two-point discrimination, receptive field, receptor spacing, resolution, two limits, model.

> **Scope of this paper (scope note)**: **No new theorem or law is claimed——that the two-point threshold differs greatly across the body, that both receptor density and receptive-field size act on it, and the way the test is set up as two peaks with a dip between them, are all known.No real values of skin are claimed——no millimetres for a fingertip or a back are given; every length here is in the units of the model.The brain is not treated——cortical magnification and somatotopy are not the subject.Absolute sensitivity is not included——a response too small to notice is not in the model; only the ratio of dip to peak is read.Methods of measurement are not discussed——staircase and constant-stimuli procedures are not entered.The sampling theorem is not restated——this is not about reconstructing a continuous image but about whether two peaks are visible.Relation to earlier papers**: **Paper 182 showed that "the curse of dimensionality" is not one curse——this paper does not sit in that seat but stands beside it: there one name pointed at several difficulties, here one threshold holds two limits.Paper 300 showed that whether two things share a root is decidable——blur and spacing are distinct roots: improving one leaves the other where it was.**What is added is **measuring the dense-array threshold at 2.286668 times σ, confirming that no criterion takes it below 2σ, measuring the floor at 1.500257 times the spacing, measuring that the floor runs from 1.003722 to 2.000256 times the spacing with the phase**, and **placing the separator on blur versus spacing**.

1. Notation and the Question

Setting: **receptors are placed along a line at spacing d, with Gaussian receptive fields.Two point stimuli sit at $\pm s/2$, and the response of receptor i is written as follows.**

$$v_i = \exp\left(-\frac{(x_i + s/2)^2}{2\sigma^2}\right) + \exp\left(-\frac{(x_i - s/2)^2}{2\sigma^2}\right), \qquad x_i = id + \varphi$$

The maxima of the left and right halves are the peaks, and the minimum between them is the dip.If the dip is below 0.95 of the peak the two points count as told apart, and the smallest such s is the threshold.The phase φ says where the two points land on the lattice, and on skin it cannot be chosen.

Question: **what sets the threshold, the width of the field or the spacing of the receptors?**

2. When the Receptors Are Dense

Computation 2.1: **with a spacing of 0.02, the field width was swept over four values (median over 24 phases).**

field width σ	threshold	threshold $/\sigma$
0.50	1.143420	2.286839
1.00	2.286690	2.286690
2.00	4.573291	2.286645
4.00	9.146553	2.286638

> **Observation 2.2**
>
> the threshold is proportional to the field width.Sweeping σ over a factor of 8 leaves a spread in threshold $/\sigma$ of only 2.008×10^{-4}, about a middle value of 2.286668——when the spacing is far finer than the width, the lattice does not enter the answer.

Computation 2.3: the dip criterion was swept (spacing 0.02, $\sigma = 1$).

how far below the peak the dip must fall	threshold $/\sigma$
0.01	2.120562
0.02	2.173903
0.05	2.286658
0.10	2.427398
0.20	2.659331

> **Observation 2.4**
>
> loosening the criterion never goes below 2σ.That is where the sum of two Gaussians first grows two peaks——the criterion moves the value of the threshold but not its floor.

3. When the Receptors Are Coarse

Computation 3.1: with the spacing fixed at 2.0, only the field was narrowed (24 phases).

field width σ	threshold (median)	smallest	largest
4.0000	9.313223	9.189187	9.374675
2.0000	4.762910	4.594594	5.129581
1.0000	3.232199	2.740282	4.068251
0.5000	3.013426	2.186110	4.012823
0.2500	3.003206	2.046528	4.003206
0.1000	3.000513	2.007445	4.000513

> **Observation 3.2**
>
> this is the core. Narrowing stops paying.Narrowing σ to a fortieth moves the threshold only from 9.313223 to 3.000513——what would have become a fortieth in a dense array stops at a 3.10th here.

> **Observation 3.3**
>
> the floor is written in the spacing.The median threshold is 1.500257 times the spacing——because a weakly responding receptor is needed between the two points.

> **Observation 3.4**
>
> the floor is not one number.With the phase it runs from 1.003722 to 2.000256 times the spacing——where the two points land on the lattice cannot be chosen, so the same skin has lucky and unlucky placements.

Computation 3.5: conversely, with the field fixed at 0.5, only the spacing was varied (median over phases).

spacing d	threshold
0.02	1.143420
0.10	1.145823
0.50	1.190728
1.00	1.616099
2.00	3.013426
4.00	6.006412

Observation 3.6

the larger of the two limits decides.Up to a spacing of 0.50 the threshold barely moves (1.143420 to 1.190728); past 1.00 it follows the spacing——the field-bound value 1.143329 and the spacing-bound value d change places where they cross.

4. The Separator

Observation 4.1

the separator is whether one asks about blur or about spacing.Narrowing the field leaves the spacing; closing the spacing leaves the field——one threshold holds two limits, and improving one alone always stops at the other.

Observation 4.2

placed beside Paper 182.182 showed that “the curse of dimensionality” is not one curse——there one name pointed at several difficulties; here one threshold holds two limits. The difference is whether the multiplicity sits in the name or in the quantity.By the criterion of Paper 300, blur and spacing are distinct roots——both raise the threshold, and moving one leaves the other where it was.

Observation 4.3

what is not said.Real values of skin, the brain, absolute sensitivity and psychophysical procedure are not treated——what is shown is that one threshold holds two limits and the larger decides.

5. Census and Closing

1. **dense gives threshold $/\sigma = 2.286668$** (Computation 2.1): **the field decides.**
2. **spread 2.008×10^{-4}** (Computation 2.1): **over a factor of 8 in σ.**
3. **no criterion goes below 2σ** (Computation 2.3): **there is a floor.**
4. **spacing 2.0, σ to a fortieth** (Computation 3.1): **the core.**
5. **threshold only to 3.000513** (Computation 3.1): **it stops at a 3.10th.**
6. **the floor is 1.500257 times the spacing** (Observation 3.3): **written in the spacing.**
7. **phase gives 1.003722 to 2.000256** (Observation 3.4): **the landing cannot be chosen.**
8. **unchanged up to spacing 0.50** (Computation 3.5): **1.143420 to 1.190728.**
9. **past 1.00 it follows the spacing** (Computation 3.5): **the larger decides.**
10. **blur, or spacing** (Observation 4.1): **the separator; Papers 182, 300.**

Closing: **Two-point discrimination has two limits: narrowing the receptive field never beats the receptor spacing.With receptors packed densely the threshold is proportional to the field width at 2.286668 times σ, with a spread of only 2.008×10^{-4} over a factor of 8 in σ, and no loosening of the dip criterion takes it below 2σ——because that is where the sum of two Gaussians first grows two peaks.Yet with the spacing fixed at 2.0, narrowing the field to a fortieth moves the threshold only from 9.313223 to 3.000513, a floor at 1.500257 times the spacing——because a weakly responding receptor is needed between the two points.That floor is not one number either: with the phase it runs from 1.003722 to 2.000256 times the spacing, and where the two points land cannot be chosen.Placed among the earlier papers——beside Paper 182's "one name, several difficulties", in the form of one threshold with two limits, and treating blur and spacing as distinct roots by the criterion of Paper 300.To be honest——every length here is in the units of the model, not millimetres of real skin.**

References

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